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USNTDC ltr, 9 Dec 1971

Technical Report: NAVTRADEVCEN 68-C-0050-1

ADVANCED SUBMARINE SYSTEMS EQUATIONS STUDY

(Final)

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November 1969

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NAVAL TRAINING DEVICE CENTER

ORLANDO. FLORIDA

Technical Report: NAVTRADEVCEN 68-C-0050-1

ADVANCED SUBMARINE SYSTEMS EQUATIONS STUDY

ABSTRACT

The goal of this study was to determine the optimum sets of equations to be used with two general types of submarine trainers. The equations of motion used were those developed by the Naval Ship Research and Development Center. They were reduced into two sets for use in training simulators, one for a basic submerged control maneuvering trainer and one for an advanced submerged control simulator to provide highly realistic ship control training through the full range of normal and casualty conditions.

The report outlines a general-purpose digital computer program, following the NSRDC standard equations, written in FORTRAN. Integration methods for digital simulators are discussed. A number of programs for testing the degree of simulation of a digital simulator program are given.

Use of these programs as applied to submarine simulation is shown with two sets of equations which eliminate 74 of the 131 coefficients used in the original NSRDC equations. The mathematical model for a submarine simulator using a very small computer is developed. A discussion of near-surface operation and wave-generation is followed by the general requirements for determination of the simulation requirements for training as opposed to research.

FOREWORD

The NSRDC (Naval Ship Research and Development Center) developed a set of equations which describes submarine motion in six degrees of freedom. The Naval Training Device Center recognized this set of equations as the "standard" set which should be utilized in submarine control trainers and on all trainer procurements required that the complete set be utilized. A medium scale digital computer is required to implement the entire set of NSRDC equations in a submarine trainer. Since the coefficients of many of the equations terms are zero or have little effect on submarine trajectories, the possibility of reducing the equations to a size which permits small scale digital computer implementation in submarine trainers was recognized by NAVTRADEVCEN.

This report, along with its companion reports, NAVTRADEVCEN 68-C-0050-2 and NAVTRADEVCEN 68-C-0050-3, is the culmination of a study by Goodyear Aerospace corporation to determine abbreviated, or optimum, equations to be used in conjunction with submarine simulators.

This report should be an invaluable aid to simulator personnel during development of submarine simulators. It presents a discussion of several factors involved in simulation, such as, integration methods, wave generation, small computer utilization and verification. It not only presents an abbreviated set of equations wherein a significant number of the original coefficients are eliminated, but presents a tool in the form of computer programs which permit further coefficient reduction investigations. Since the required complexity of equations are directly proportional to the training device requirements, the programs are written so that the investigator may taylor the coefficients to reach any desired level of simulation.

NAVTRADEVCEN 68-C-0050-2 presents descriptions, flow charts and listings of each computer program and NAVTRADEVCEN 68-C-0050-3 presents results of the computer programs using the SS(N)594 submarine as the demonstration model.

CHARLES A. RUMBOUGH
Project Engineer

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SECTION I

INTRODUCTION

There are two general types of submarine trainers: (1) The basic high speed submarine trainer; and (2) The advanced submerged control trainer. The first type, which does not necessarily duplicate a specific class of submarine, is used to provide initial submarine control indoctrination to potential planesmen and diving officers. The second type provides highly-realistic ship control training, which extends over the operating range of the submarine to ship control personnel. Both types of trainers utilize computers, programed with equations of motion, to simulate submarines under various operating conditions.

In the past, the equations used in these programs were somewhat arbitrarily chosen by manufacturers of training devices. Then an attempt was made by the Naval Ship Research and Development Center[1]* toward standardizing the equations.

This brings us to the purpose of this study:

1. To determine an optimum set of equations of motion to be used with each of the two general types of submarine trainers.

2. To write the two sets of equations as compactly as possible in order that they may be used with a small, general-purpose computer.

Two additional reports resulted from this contract and are described as follows:

Report Number	Remarks
68-C-0050-3	Report is CONFIDENTIAL and titled, "Advanced Submarine Systems Equations Test Data (U)".
68-C-0050-2	Report is UNCLASSIFIED and titled, "Advanced Submarine Systems Programming".

*Superscript numbers indicate references.

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SECTION II

STATEMENT OF THE PROBLEM

The determination of optimum sets of equations to be used with the two general types of submarine control trainers forms the problem. The tasks required to solve this problem are the determination of the simulation requirements. Application of these requirements in an analysis of the equations of motion, programing the equations of motion on a digital computer, and then verification that any changes to optimize them do not exceed the limits specified for good simulation.

A. PROGRAMING THE GENERAL NSRDC EQUATIONS

The problem analysis is based on the assumption that the NSRDC equations are accurate for the simulation of operational submarine motion. Therefore, the first task is to program these equations so that a digital program representing the complete equations of motion is available. The most practical programing is achieved with the use of a problem-oriented language (FORTRAN) so that the programs are independent of the computer used. This will allow the output of the study to be used by any researcher interested in submarine simulation on any computer equipped with a FORTRAN compiler.

Because this program is going to be a research tool, it is necessary to build in the greatest degree of flexibility possible. Therefore, consideration must be given to allowing any changes in parameters, variables and analytical methods within the framework of the basic equations without having to recompile subroutines at each run.

The final result should be a program that will input coefficients easily, produce any outputs desired, and will make a permanent record of any changes. Also, it should be written so that many runs can be made without having to start the entire program over again.

The next problem is to verify the research program against the available data on submarine performance to determine the accuracy of the program. A complete set of runs, following established submarine test practice, are run using the NSRDC coefficients, and these are compared with the NSRDC reports on a specific demonstration submarine. These runs verify the program and establish proper response in various input conditions and also establish a data base with which the reduced programs can be compared. In effect, a test guide is completed, and any approximations made to the program must be justified against the base program.

B. MODIFIED AND BASIC EQUATIONS

After the research program is operational, it will be necessary to establish the requirements for the basic and modified programs. The first step will be to analyze the existing performance data on submarines and the use of this data in present submarine simulators. The main sources

of data will be the NSRDC reports and the dynamic reports on present day trainers. Also, a number of studies made in the past will have to be reviewed.

A reduction to a basic equation set can be made after the simulation limits are set. The coefficients can be divided into broad categories and the effect of eliminating certain ones can be studied. Computer runs are used to aid the simplifications of the coefficient terms. For example, if it is desired to know the effect of a cross-coupling term, runs will be performed with and without the coupling term in question. The importance of the term can easily be established when the two runs are compared.

The above procedure can indicate the effect of any change, but it does not indicate the effect of response on the trainee when he is part of the closed-loop controlling the simulator. This is one of the hardest areas to cover in a study of this type because closed-loop real-time operation cannot be run on a typical batch process digital computer operation. An autopilot will be programed as part of the longitudinal loop to keep the submarine on a level course when turns are made. Measurement of the longitudinal parameters over time as the autopilot corrects for effects of lateral displacement can be used to determine the nature and quality of dynamic responses. This procedure is far more effective than inserting step inputs and measuring the output after a time delay.

After a complete analysis, the two sets of equations will cover a basic submerged control trainer and a complete simulation of a particular submarine. These equations will serve as both a demonstration of data application to a particular simulator and as a reference for any submarine trainer or simulator in the future.

C. ANALYSIS OF SURFACE CONDITIONS

The coefficients used for the analysis are deep-surface coefficients and are not applicable to a submarine moving close to the surface. The change in the coefficients due to the presence of a free surface can be calculated by hydrodynamic methods, and such a set of coefficients has been generated by NSRDC. This set is used in the research program with values adjusted for different depths and the resulting given frequencies compared with the deep submergence set to determine the surface effect on the submarine. An analysis will indicate the importance of any change.

The biggest disturbing effect of a submarine operating close to the surface is that of waves breaking overhead. Near-surface operation is becoming more important, so a good wave simulation is essential for a modern submarine simulator.

Wave action can be described only on a statistical basis because the sea surface is never regular. It is a jumble of different waves of different heights all changing and all moving in different directions. This action can be described by means of spectrum analysis and the problem to be solved is how to apply this technique to a pratical submarine simulator.

The height of waves above the submarine is the first thing that has to be known. This is usually specified by a one-dimensional spectrum, particularly that of Neumann which was presented in 1953. At least three other spectral densities have been used since then, so the first task of this part of the problem was to determine which of the various spectral densities have the most suitable form for both submarine response and for ease of computation with a small digital computer. Enough background will be given to show how any spectrum can be used in a wave generator, and one will be chosen and programed to illustrate the method on a practical basis.

The addition of a direction function to the wave spectrum is necessary because the difference between direction of ship travel and the direction of wave propagation has a direct bearing on the forces felt by a submerged submarine. Therefore, part of the solution will be to develop this directional function and show how it is applied. The calculated forces and moments will be determined to give a realistic motion under all conditions of wave inputs. A very large number of papers and references have been published in this area in the past few years. These will be surveyed and a list drawn up that will enable future workers to bring themselves up to date quickly on the present state-of-the-art in wave generation.

D. SMALL COMPUTER ANALYSIS

The last problem to be investigated once the program limits have been determined is that of a small computer analysis.

When the modified and basic program outlines have been set and a maximum reduction has been made, it is possible to determine the necessary core size to hold each of the programs. The analysis should try to reduce this factor as much as possible because it is directly proportional to the cost of the computer being used.

The programs are then analyzed from the total number of tasks to be performed. This includes not only the arithmetic operations but also all input-output subroutines and any other bookkeeping operations necessary to make up the complete cycle. All operations are sorted by category and time to determine the necessary computer speed for real-time operation. Finally, the reduced programs will be run on a 16-bit computer and compared against the programs run on the IBM 360 32-bit computer.

A small computer survey and state-of-the-art review will be made as of the date of this study. This review will be out of date soon at the rate at which the computer industry is advancing, but it will serve to indicate an idea of what can be done in future submarine simulators.

SECTION III

METHODS AND PROCEDURES

A. PROGRAMING THE NSRDC EQUATIONS

The NSRDC standard equations of motion for submarine simulation cover all phases of submarine simulation in six-degrees-of-freedom, including emergency recoveries after casualties. These equations come from NSRDC Report 2510 which contains a brief history, defines the mathematical model, discusses the coefficients required, and sets a standard to be used in the simulation of submarines. These equations were programed in two forms. They are given in program EB920 - Submarine Simulation, and ZC790 - Submarine Simulation, Longitudinal Freedom Only found in the programing report, NAVTRADEVCEN 68-C-0050-3.

EB920 is a research submarine simulation program that allows the simulation of any submarine on a large general-purpose digital computer. It is complete and flexible in that all NSRDC coefficients are programed, and changes can be made easily between runs.

Figure 1 is a block diagram of the program with all subroutines shown. The program operates as follows.

The main submarine simulation program ties the various subroutines together and determines the operating limits placed on each run. The first call is to INPUT. This subroutine reads all coefficients from a data deck along with controls to set the output, determines which integration subroutine to use, and which control input schedule should be used. The main program then writes out all coefficients so that the input to the run has a permanent record printed at the start. An integration switch then calls either the KUTTA or INTEG subroutine. KUTTA is a very accurate method of integration that is used as a standard for comparing changes between runs. INTEG has three short methods of integration built in and can be easily adapted so that any integration method can be used without changing the rest of the program.

Depending on which integration subroutine is used, EVAL or EVAL 1 is called. They both compute the equations of motion as given in equations (1) through (7) from NSRDC Report 2510[1]. EVAL has matrix invert and multiply subroutines so that the equations can be solved exactly in each cycle. EVAL 1 uses values computed on the last cycle in some cases. Both subroutines call CONTR which contains schedules for a number of inputs representing the deflection of the controls on the actual submarine. A return to INTEG integrates all parameters and the cycle is finished. The main program then prints out the twelve variables of interest on demand and checks the RUN TIME to see if the program is finished.

Plotting calls have been added to the main program so that a plot can be made of any of the twelve output variables and surface control deflections versus time. These plots can be made in any order and the
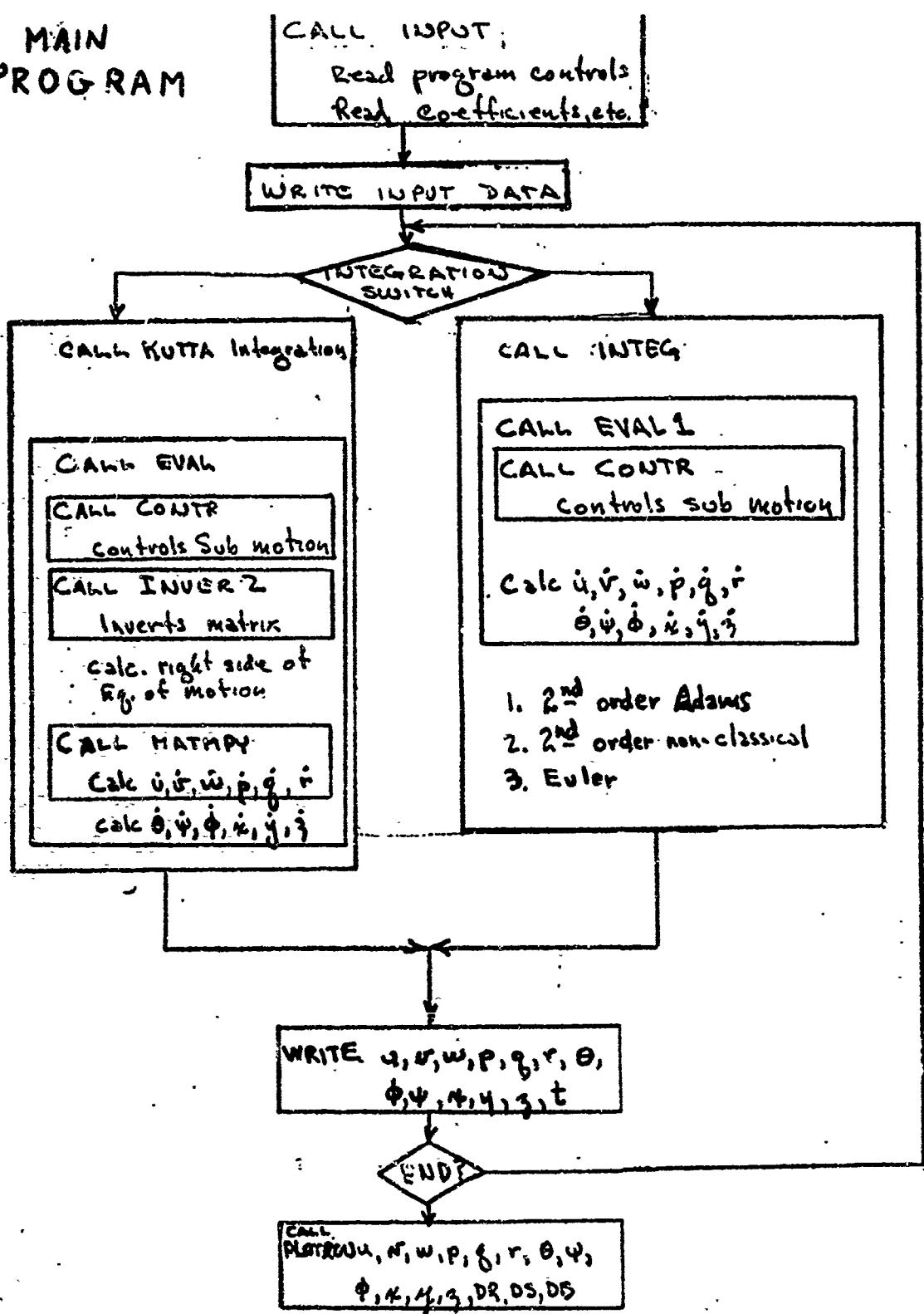
Figure 1. Research Simulation Program Block Diagram

AXIAL FORCE

$$m\left[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})\right] =$$

$$+ \frac{\rho}{2}\ \ell^4\left[X_{qq}'\ q^2 + X_{rr}'\ r^2 + X_{rp}'\ rp\right]$$

$$+ \frac{\rho}{2}\ \ell^3\left[X_{\dot{u}}'\ \dot{u} + X_{vr}'\ vr + X_{wq}'\ wq\right]$$

$$+ \frac{\rho}{2}\ \ell^2\left[X_{uu}'\ u^2 + X_{vv}'\ v^2 + X_{ww}'\ w^2\right]$$

$$+ \frac{\rho}{2}\ \ell^2 u^2\left[X_{\delta r\ \delta r}'\ \delta r^2 + X_{\delta s\ \delta s}'\ \delta s^2 + X_{\delta b\ \delta b}'\ \delta b^2\right]$$

$$+ \frac{1}{2}\ \rho \ell^2\left[a_i\ u^2 + b_i\ uu_c + c_i\ u_c^2\right]$$

$$- (W - B)\ \sin\theta$$

$$+ \frac{\rho}{2}\ \ell^2\left[X_{vv\eta}'\ v^3 + X_{ww\eta}'\ w^3 + X_{\delta r\delta r\eta}'\ \delta_r^2\ u^2\right.$$

$$\left. + X_{\delta s\delta s\eta}\ \delta_s^2\ u^2\right](\eta - 1)$$

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LATERAL FORCE

$$m\left[\dot{v} - wp + ur - y_G\left(r^3 + p^2\right) + z_G\left(qr - \dot{p}\right) + x_G\left(qp + \dot{r}\right)\right] =$$

$$+\frac{\rho}{2}\ell^4\left[Y_{\dot{r}}{}'\,\dot{r} + Y_{\dot{p}}{}'\,\dot{p} + Y_{p|p|}{}'\,p|p| + Y_{pq}{}'\,pq + Y_{qr}{}'\,qr\right]$$

$$+\frac{\rho}{2}\ell^3\left[Y_{\dot{v}}{}'\,\dot{v} + Y_{vq}{}'\,vq + Y_{wp}{}'\,wp + Y_{wr}{}'\,wr\right]$$

$$+\frac{\rho}{2}\ell^3\left[Y_r{}'\,ur + Y_p{}'\,up + Y_{|r|\delta r}{}'\,u|r|\delta r + Y_{v|r|}{}'\,\frac{v}{|v|}\left|\left(v^2 + w^2\right)^{\frac{1}{2}}\right|\,|r|\right]$$

$$+\frac{\rho}{2}\ell^2\left[Y_*{}'\,u^2 + Y_v{}'\,uv + Y_{v|v|}{}'\,v\left|\left(v^2 + w^2\right)^{\frac{1}{2}}\right|\right]$$

$$+\frac{\rho}{2}\ell^2\left[Y_{vw}{}'\,vw + Y_{\delta r}{}'\,u^2\,\delta r\right]$$

$$+ (W - B)\cos\theta\sin\phi$$

$$+\frac{\rho}{2}\ell^3\,Y_{r\eta}{}'\,ur\,(\eta - 1)$$

$$+\frac{\rho}{2}\ell^2\left[Y_{v\eta}{}'\,uv + Y_{v|v|\eta}{}'\,v\left|\left(V^2 + w^2\right)^{\frac{1}{2}}\right| + Y_{\delta r\eta}{}'\,\delta_r\,u^2\right](\eta - 1)$$

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NORMAL FORCE

$$m\left[\dot{w} - uq + vp - z_G\,(p^2 + q^2) + x_G\,(rp - \dot{q}) + y_G\,(rq + \dot{p})\right] =$$

$$+\frac{\rho}{2}\,\ell^4\left[Z_{\dot{q}}{}'\,\dot{q} + Z_{pp}{}'\,p^2 + Z_{rr}{}'\,r^2 + Z_{rp}{}'\,rp\right]$$

$$+\frac{\rho}{2}\,\ell^3\left[Z_{\dot{w}}{}'\,\dot{w} + Z_{vr}{}'\,vr + Z_{vp}{}'\,vp\right].$$

$$+\frac{\rho}{2}\,\ell^3\left[Z_q{}'\,uq + Z_{|q|\delta s}{}'\,u|q|\delta s + Z_{w|q|}{}'\,\frac{w}{|w|}\left|(v^2 + w^2)^{\frac{1}{2}}\right|\,|q|\right]$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_*{}'\,u^2 + Z_w{}'\,uw + Z_{w|w|}{}'\,w|(v^2 + w^2)^{\frac{1}{2}}|\right].$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_{|w|}{}'\,u|w| + Z_{ww}{}'\,|w\,(v^2 + w^2)^{\frac{1}{2}}|\right]$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_{vv}{}'\,v^2 + Z_{\delta s}{}'\,u^2\,\delta s + Z_{\delta b}{}'\,u^2\,\delta b\right]$$

$$+ (W - B)\cos\theta\cos\phi$$

$$+\frac{\rho}{2}\,\ell^3\,Z_{q\eta}{}'\,uq\,(\eta - 1)$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_{w\eta}{}'\,uw + Z_{w|w|\eta}{}'\,w|(v^2 + w^2)^{\frac{1}{2}}| + Z_{\delta s\eta}{}'\,\delta_s u^2\right](\eta - 1)$$

ROLLING MOMENT

$$I_x \dot{p} + (I_z - I_y) qr - (\dot{r} + pq) I_{xz} + (r^2 - q^2) I_{yz} + (pr - \dot{q}) I_{xy}$$

$$+ m \left[y_G (\dot{w} - uq + v\rho) - z_G (\dot{v} - wp + ur) \right] =$$

$$+ \frac{\rho}{2} \ell^5 \left[K_{\dot{p}}' \dot{p} + K_{\dot{r}}' \dot{r} + K_{qr}' qr + K_{pq}' pq + K_{p|p|}' p|p| \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[K_p' up + K_r' ur + K_{\dot{v}}' \dot{v} \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[K_{vq}' vq + K_{wp}' wp + K_{wr}' wr \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[K_*' u^2 + K_v' uv + K_{v|v|}' v|(v^2 + w^2)^{\frac{1}{2}}| \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[K_{vw}' vw + K_{\delta r}' u^2 \delta r \right]$$

$$+ (y_G W - y_B B) \cos \theta \cos \phi - (z_G W - z_B B) \cos \theta \sin \phi \Big]$$

$$+ \frac{\rho}{2} \ell^3 K_{*\eta}' u^2 (\eta - 1)$$

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PITCHING MOMENT

$$I_y \dot{q} + (I_x - I_z) rp - (\dot{p} + qr) I_{xy} + (p^2 - r^2) I_{zx} + (qp - \dot{r}) I_{yz}$$

$$+ m \left[z_G (\dot{u} - vr + wq) - x_G (\dot{w} - uq + vp) \right] =$$

$$+ \frac{\rho}{2} \ell^5 \left[M_{\dot{q}}{}' \dot{q} + M_{pp}{}' p^2 + M_{rr}{}' r^2 + M_{rp}{}' rp + M_{q|q|}{}' q|q| \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[M_{\dot{w}}{}' \dot{w} + M_{vr}{}' vr + M_{vp}{}' vp \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[M_q{}' uq + M_{|q|\delta s}{}' u|q|\delta s + M_{|w|q}{}' |(v^2 + w^2)^{\frac{1}{2}}| q \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{*}{}' u^2 + M_w{}' uw + M_{w|w|}{}' w |(v^2 + w^2)^{\frac{1}{2}}| \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{|w|}{}' u|w| + M_{ww}{}' |w (v^2 + w^2)^{\frac{1}{2}}| \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{vv}{}' v^2 + M_{\delta s}{}' u^2 \delta s + M_{\delta b}{}' u^2 \delta b \right]$$

$$- (x_G W - x_B B) \cos \theta \cos \phi - (z_G W - z_B B) \sin \theta$$

$$+ \frac{\rho}{2} \ell^4 M_{q\eta}{}' uq (\gamma - 1)$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{w\eta}{}' uw + M_{w|w|\eta}{}' w|(v^2 + w^2)^{\frac{1}{2}}| + M_{\delta s\eta}{}' \delta_s u^2 \right] (\eta - 1)$$

YAWING MOMENT

$$I_z \, \dot{r} + (I_y - I_x) \, pq - (\dot{q} + rp) \, I_{yz} + (q^2 - p^2) \, I_{xy} + (rq - \dot{p}) \, I_{zx}$$

$$+ m \left[x_G \, (\dot{v} - wp + ur) - y_G \, (\dot{u} - vr + wq) \right] =$$

$$+ \frac{\rho}{2} \, \ell^5 \left[N_{\dot{r}}{}' \, \dot{r} + N_{\dot{p}}{}' \, \dot{p} + N_{pq}{}' \, pq + N_{qr}{}' \, qr + N_{r|r|}{}' r|r| \right]$$

$$+ \frac{\rho}{2} \, \ell^4 \left[N_{\dot{v}}{}' \, \dot{v} + N_{wr}{}' \, wr + N_{wp}{}' \, wp + N_{vq}{}' \, vq \right]$$

$$+ \frac{\rho}{2} \, \ell^4 \left[N_{p}{}' \, u\dot{p} + N_{r}{}' \, ur + N_{|r|\delta r}{}' \, u|r|\delta r + N_{|v|r}{}' \, |(v^2 + w^2)^{\frac{1}{2}} | r \right]$$

$$+ \frac{\rho}{2} \, \ell^3 \left[N_{*}{}' \, u^2 + N_{v}{}' \, uv + N_{v|v|}{}' \, v \, |(v^2 + w^2)^{\frac{1}{2}} | \right]$$

$$+ \frac{\rho}{2} \, \ell^3 \left[N_{vw}{}' \, vw + N_{\delta r}{}' \, u^2 \delta r \right]$$

$$+ (x_G \, W - x_B \, B) \cos \theta \, \sin \phi + (y_G \, W - y_B \, B) \sin \theta$$

$$+ \frac{\rho}{2} \, \ell^4 \, N_{r\eta}{}' \, ur \, (\eta - 1)$$

$$+ \frac{\rho}{2} \, \ell^3 \left[N_{v\eta}{}' \, uv + N_{v|v|\eta}{}' \, v|(v^2 + w^2)^{\frac{1}{2}} | + N_{\delta r\eta}{}' \, \delta_r \, u^2 \right] (\eta - 1)$$

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KINEMATIC RELATIONS (7)

$$U^2 = u^2 + \mathcal{N}^2 + \omega^2$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

$$\dot{\psi} = \frac{q\sin\phi + r\cos\phi}{\cos\theta}$$

$$\dot{\phi} = p + \dot{\psi}\sin\phi$$

$$\dot{x}_0 = u\cos\theta\cos\psi + \mathcal{N}(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi)$$

$$+ \omega(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)$$

$$\dot{y}_0 = u\cos\theta\sin\psi + \mathcal{N}(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi)$$

$$+ \omega(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)$$

$$\dot{z}_0 = -u\sin\theta + \mathcal{N}\cos\theta\sin\phi + \omega\cos\theta\cos\phi$$

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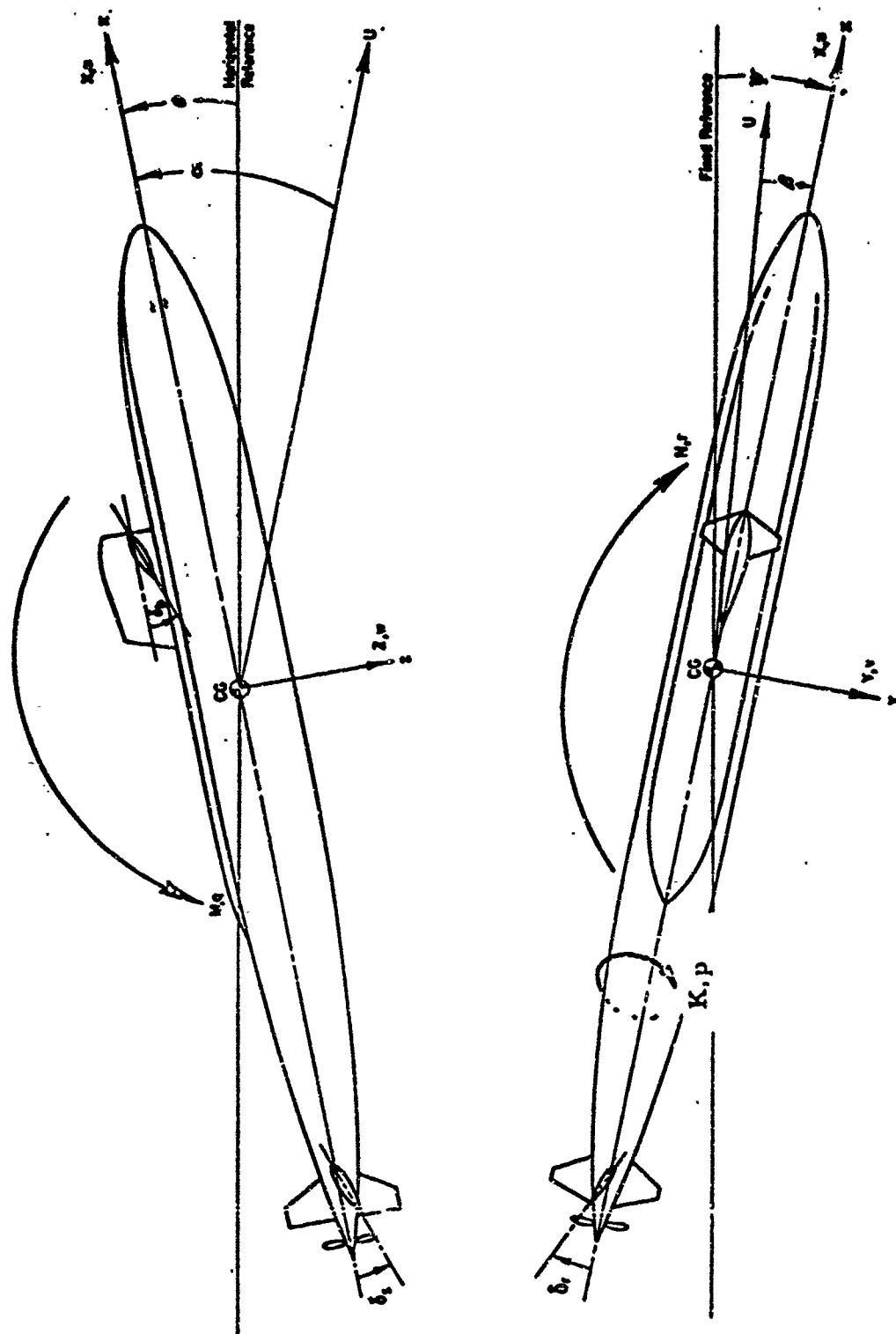
Figure 2. Positive Directions of Axes and Variables

corre t labeling will be added to the plot. However, program runs do not depend on the plotting subroutines, so if a plotter is not available, the printed output serves in place of the plots.

Almost all variables, in addition to the coefficients, are located in the COMMON area of the program. This allows a degree of flexibility that could not be obtained otherwise and the main program takes advantage of this to make changes between runs. After a run is finished, all values are restored to their original values at the start of the run. The next cards in the input deck are then checked and a coding system allows any variable in COMMON to be changed to a new value. Integration methods and intervals, input schedules, and output requirements as well as the coefficients can be changed between each run. Because of the heavy use of COMMON, details are given in the Programing Report.

ZC790 is identical to EB920 except that only three-degrees-of-freedom are simulated. Only the longitudinal mode is active so all terms multiplied by p, v, or r are set to zero and removed. The block diagram is shown in figure 3. Only one method of integration is supplied. The CONTR subroutine has removed all provisions for rudder motion. Therefore, only meander, overshoot, and similar runs can be calculated. This program was written to save computer time because it runs in one-third the time of EB920. This is a real advantage where a large number of longitudinal runs are required. About three-fourths of the total runs made during the course of this study were of this type. The mathematical model for program ZC790 is given in equations (8) through (11).

The mathematical model for the simulation program used two different sets of notation because of the FORTRAN restriction that required the use of capital letters only. Therefore, the following notation is adapted and carried throughout this study. The letters X, Y, Z, K, M, and N stand for coefficients in the axial, etc., channels respectively. U, V, W, P, Q, and R stand for the velocities, and whenever an absolute value is stated, the velocity will be preceeded by an A. Accelerations are velocities followed by a D. The surfaces are preceeded by a D for delta and eta terms are followed by an E. Inertias and other terms are similarly identified. For example,

$$X_{rp} \approx XRP$$
$$Y_{|r|\delta r} \approx YARDR$$
$$Z_{\dot{w}} \approx ZWD$$
$$M_{\delta b} \approx MDB$$
$$N_{|v|v|\eta} \approx NAVAVE$$
$$I_{xy} \approx IXY$$

B. INTEGRATION METHODS

This section discusses some of the many integration methods that can be used for digital computation, why they were chosen to implement the equations of motion for submarines, and how they are utilized in the submarine simulation. The optimum integration step-size and a stability criterion for any integration method is discussed also.

MAIN
PROGRAM

CALL INPUT
  Read program controls
  Read coefficients, etc

WRITE INPUT DATA

CALL INTEG

CALL EVAL1

CALL CONTR
  Controls sub motion

Calc $\dot{u}$, $\dot{\omega}$, $\dot{q}$, $\dot{\theta}$, $\dot{N}$, $\dot{x}$

2nd Order Adams Integration

WRITE $u, x, \omega, \theta, q, t, \theta$

END?

CALL PLOTROU (Plot) $u, x, \omega, \theta, q, t, \theta$, DS, DB
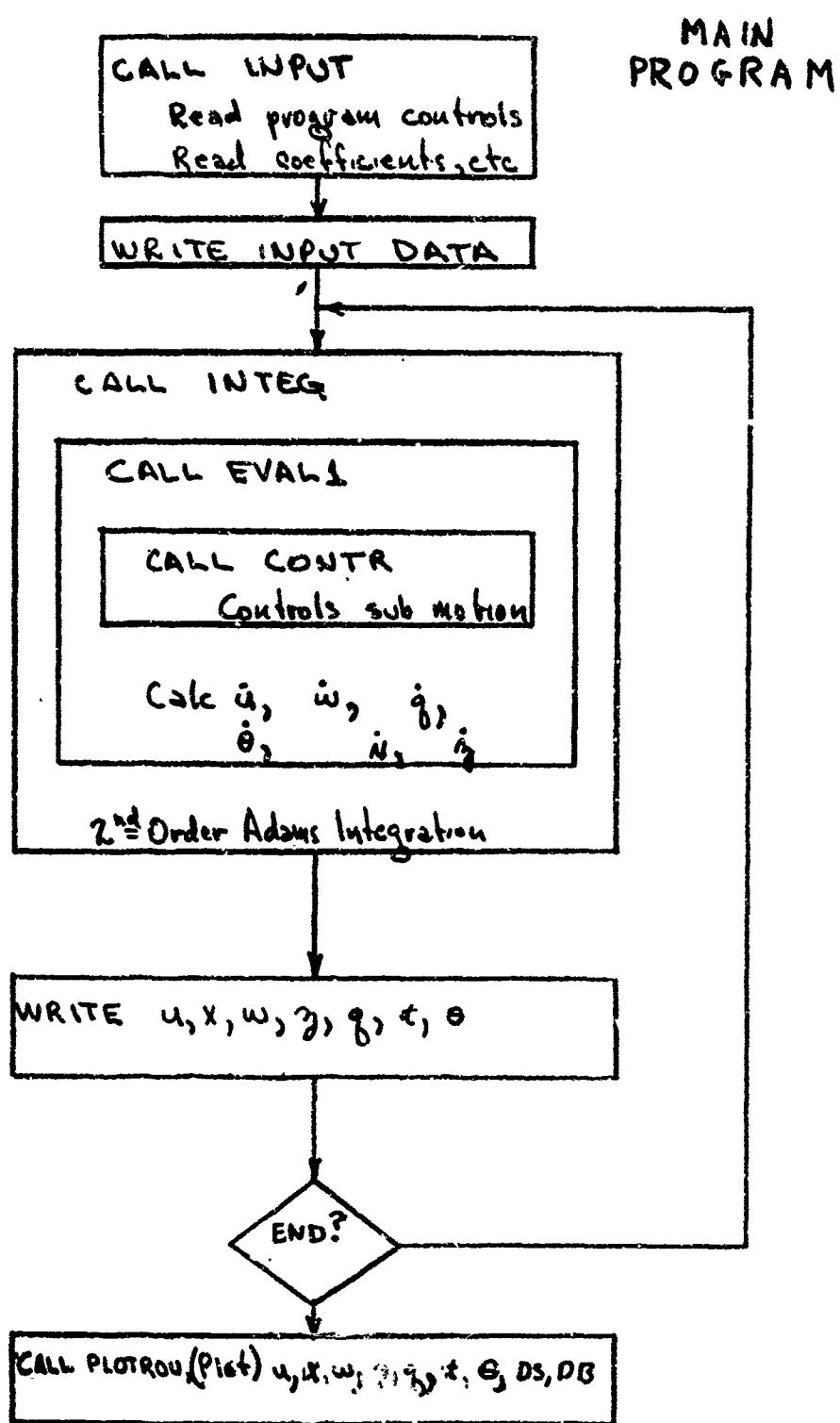
Figure 3. Research Simulation Program, Longitudinal Freedom,
Block Diagram

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Equations of Motion for Longitudinal
Freedom Only (ZC790 Math Model)

AXIAL FORCE (8)

$$m\left[\dot{u} + \omega q - X_{q} q^2 + Z_{q} \dot{q}\right] =$$

$$+ \frac{\rho}{2} \ell^4 \left[X_{qq}' q^2\right]$$

$$+ \frac{\rho}{2} \ell^3 \left[X_{\dot{u}}' \dot{u} + X_{\omega q}' \omega q\right]$$

$$+ \frac{\rho}{2} \ell^2 \left[X_{uu}' u^2 + X_{\omega\omega}' \omega^2\right]$$

$$+ \frac{\rho}{2} \ell^2 u^2 \left[X_{\delta s \delta s}' \delta s^2 + X_{\delta b \delta b}' \delta b^2\right]$$

$$+ \frac{\rho}{2} \ell^2 \left[a_i u^2 + b_i u u_c + c_i u_c^2\right]$$

$$- (W - B) \sin\theta$$

$$\cdot \frac{\rho}{2} \ell^2 \left[X_{\omega\omega\eta}' \omega^2 + X_{\delta s \delta s \eta}' \delta_s^2 u^2\right](\eta - 1)$$

KINEMATIC RELATIONS (9)

$$U^2 = u^2 + \omega^2$$

$$\dot{\theta} = q$$

$$\dot{Z}_o = \omega\cos\theta - u\sin\theta$$

$$\dot{X}_o = u\cos\theta + \omega\sin\theta$$

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PITCHING MOMENT (10)

$$I_y \dot{q} + m\left[Z_G(\dot{u} + wq) - X_G(\dot{w} - uq)\right] =$$

$$\frac{\rho}{2} \ell^5 \left[M_{\dot{q}} \dot{q} + M_{q'|q'|} q'|q'|\right]$$

$$+ \frac{\rho}{2} \ell^4 \left[M_{\dot{w}} \dot{w}\right]$$

$$+ \frac{\rho}{2} \ell^4 \left[M_q \, uq + M_{|q|\delta_s} u|q|\delta_s + M_{|w|q} |w| q\right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_* u^2 + M_w \, uw + M_{w|w|} w|w|\right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{|w|} u|w| + M_{ww|w|} w|w|\right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{\delta_s} u^2 \delta_s + M_{\delta_b} u^2 \delta_b\right]$$

$$- (X_G W - X_B B) \cos\theta - (Z_G W - Z_B B) \sin\theta$$

$$+ \frac{\rho}{2} \ell^4 M_{q\eta} \, uq(\eta - 1)$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{w\eta} uw + M_{w|w|\eta} w|w| + M_{\delta_s\eta} \delta_s u^2\right](\eta - 1)$$

NORMAL FORCE (11)

$$m\left[\dot{w} - uq - Z_G q^2 - X_G \dot{q}\right] =$$

$$+ \frac{\rho}{2}\ell^4\left[Z_{\dot{q}}'\dot{q}\right]$$

$$+ \frac{\rho}{2}\ell^3\left[Z_{\dot{w}}'\dot{w}\right]$$

$$+ \frac{\rho}{2}\ell^3\left[Z_q'uq + Z_{|q|\delta s}'u|q|\delta_s + Z_{w|q|}'w|q|\right]$$

$$+ \frac{\rho}{2}\ell^2\left[Z_*'u^2 + Z_w'uw + Z_{w|w|}'w|w|\right]$$

$$+ \frac{\rho}{2}\ell^2\left[Z_{|w|}'u|w| + Z_{ww}'|w|w|\right]$$

$$+ \frac{\rho}{2}\ell^2\left[Z_{ss}'u^2\delta_s + Z_{sb}'u^2\delta_b\right]$$

$$+ (W - B)\cos\theta$$

$$+ \frac{\rho}{2}\ell^3\left[Z_{q\eta}'uq\right](\eta - 1)$$

$$+ \frac{\rho}{2}\ell^2\left[Z_{w\eta}'uw + Z_{w|w|\eta}'w|w| + Z_{ss\eta}'\delta_s u^2\right](\eta - 1)$$

There are a great number of integration methods that can be used
in digital computation. Some of these methods have been in use for many
years. They were developed to integrate functions numerically that could
not be solved in closed form. Others have been developed fairly recently
to take advantage of the digital computer's high speed and ability to handle
large blocks of numbers. They range all the way from the simple Euler's
method to a fourteen-step predictor-corrector developed by NASA to calculate
the position of the planets with great accuracy.

The choice of any integration method is determined by considering the
trade-off between accuracy, stability, and speed. The accuracy is depend-
ent on the amount of truncation and round-off errors present when using a
particular method. Truncation error results from the fact that digital
integration is a discrete process using a finite number of equally spaced
points rather than a series with an infinite number of terms. Accuracy
can be improved by using more points as a past history or by making the
integration step interval smaller. Either method buys increased accuracy
at the cost of speed. Round-off error results when generating a sequence
of numbers to represent the integrated function. Each number is limited
by the size of the computer word being used. However, there are methods
for determining an upper bound on the magnitude of round-off errors ex-
pected for the numerical accuracy being used. Studies have shown that
for five decimal figure accuracy within the limits of a sixteen-bit
machine, round-off errors are of no consequence compared to the expected
truncation errors.[2]

Stability of an integration method is defined as the ability to
control propagated errors so that they do not increase as time passes.
Propagated errors are the sum of both truncation and round-off errors
over all integration steps calculated since the problem started. The
amount of propagated error is the single most important criteria in the
choice of any integration method for simulation. Fortunately, there is
a comparatively easy procedure for determining if a particular integration
method can be used with a particular simulation problem. This procedure
will be explained below using the submarine simulation as an example.

Speed depends on the complexity of the method used and the size of
the integration interval. Real-time simulation requires fitting all
computations for one cycle within the integration interval while still
keeping the desired accuracy. Therefore, this trade-off has to be made
for each simulation. In the case of this study it was proven that the
problem was not critical for submarines.

Integration methods can be divided into CLOSED and OPEN classes.
If a rate at some specified time is required to calculate a position,
the scheme is said to be a CLOSED (or corrector) formula. If the rate
is not required, the scheme is an OPEN (or predictor) one. Closed
methods cannot be used to integrate the equations of motion because the
rates are a function of the position at that time. Since the positions
are the output of the calculations integrated from the rates, the rates
are not known. On the other hand, open formulas use rates calculated in
prior integration cycles so that the present rates are not required. The

last rates are used in the open integration formulas to give the present position and the new positions are used to calculate the present rates from the equations of motion. In order to keep the different methods straight, a notation has been developed in which the open and closed formulas are specified by Onm and Cnm respectively, and n is the number of former positions used and m is the number of former rates used. The difference equations expressing the integrated position as a function of rates and position is

$$X_n = \sum_{i=1}^{N} a_i X_{n-i} + h \sum_{i=0}^{M} b_i \dot{X}_{n-i}$$

where i is the discrete time at which a calculation has been made in the past and n is the present time. The a's and b's are parameters which determine the particular integration method. H is the integration interval or step size and has the dimension of time in order to keep the dimensions consistent across the equation. Note that if b_o is zero, then $\dot{X}_n$ is not needed to calculate X_n and the system is an open or predictor method. Therefore, for simulation work, b_o should be zero.

For an example, let us look at the 2nd order Adams method of integration which is an O_{12} type. In this case, $a_1 = 1, b_o = 0, b_1 = 3/2$, and $b_2 = -\frac{1}{2}$. The difference equations would be

$$X_n = X_{n-1} + h\left(\frac{3}{2}\dot{X}_{n-1} - \frac{1}{2}\dot{X}_{n-2}\right)$$

Many different integration formulas have been extensively studied and almost any text on numerical methods goes into them in some detail.[3]

The problem was to determine which of these methods should be used for a submarine simulator and which one should be used for the research program. Finding an integration method for the research program was comparatively easy. Since the program did not have to run in real-time, the only criterion that had to be met was to have the most accurate program available.

The method chosen is a four step Runge-Kutta predictor-corrector. It was stated that corrector (closed) methods could not be used in simulation, but in this case two difference equations are used. An open equation is used to predict a position and the closed equation is used to check this calculation. The sequence is as follows. A new position is integrated from the prior known rates using an open formula. This new position is used to calculate a new rate from the equations of motion. The new position and new rate can now be used in a closed formula to integrate a new corrected position. The new corrected position is used in the equations of motion to calculate a new corrected rate. This process is then repeated. This is enough to reduce the error between two successive position calculations to a very small amount.

The exact difference equations are not listed here but are given in the programming report under the KUTTA subroutine.

The next task was to determine which integration method should be

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used in a submarine simulator. It is obvious that speed and simplicity are desired, provided that accuracy can be maintained. A number of studies have been performed using different techniques for integration in flight simulation and, based on these reports, two different methods were chosen for examination against the Runge-Kutta method.

The simplest method of all is an O_{11} due to Euler with the following difference equation.

$$X_n = X_{n-1} + h \dot{X}_{n-2}$$

The last calculated rate is multiplied by the integration step size and is then added to the position known up to that time to give the new position. This difference equation has a bad reputation. In fact, Jennings[3] says that "it is a naive method of very limited accuracy and not recommended in practice". However, it has only one multiplication, one addition, and no storage is required to hold former values. This makes it the shortest, and therefore the fastest, of any method and since one of the objectives of this study is to find the smallest total program this method was examined.

In 1965 Goodyear Aerospace Corp. performed an in-house research and development study[5], utilizing their simulation laboratory digital computer facility, to investigate a number of complex integration algorithms used for digital computers in general to see if such complexity was needed for flight simulation. At that time, it was proposed that digital flight simulators use such formulas as Hammings $O_{12} C_{32}$ or the $O_{30} C_{31}$ Mod Gurk developed at the University of Pennsylvania. The conclusions of this study were that excellent results, well within the tolerance allowed in the simulation of aircraft systems, can be obtained with much simpler algorithms. It was found that the smallest errors occurred among the simpler routines with the 2nd order Adams difference equation

$$X_n = X_{n-1} + \frac{h}{2}\left(3\dot{X}_{n-1} - \dot{X}_{n-2}\right)$$

This requires an additional subtraction and a storage for each parameter to be integrated. Since this one has been used very successfully at Goodyear Aerospace, and Euler integration has larger errors, 2nd order Adams was also implemented in the subroutines.

The next step was to look at the way integrations are handled in the NSRDC equations of motion. The six-degree-of-freedom problem requires that twelve integrations be performed, because the second order equations have been expressed as a system of simultaneous first order equations. Using matrix notation

$$[M][\dot{X}] = [X]$$

where $[\dot{X}]$ are the rates and $[X]$ are the coefficient summations. The matrix $[M]$ comes from the fact that the rates are functions of other rates and therefore have to be solved for simultaneously. If $[\dot{X}] = [\dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}]^T$, then this matrix is

$$\begin{bmatrix} m-\frac{P}{2}\ell^3 X_{\dot u} & 0 & 0 & 0 & mZ_G & -mY_G \\ 0 & m-\frac{P}{2}\ell^3 Y_{\dot v} & 0 & -\frac{P}{2}\ell^4 Y_{\dot p}-mZ_G & 0 & -\frac{P}{2}\ell^4 Y_{\dot r}+mX_G \\ 0 & 0 & m-\frac{P}{2}\ell^3 Z_{\dot w} & mY_G & -\frac{P}{2}\ell^4 Z_{\dot q}-mX_G & 0 \\ 0 & -\frac{P}{2}\ell^4 K_{\dot v}-mZ_G & mY_G & I_X-\frac{P}{2}\ell^5 K_{\dot p} & -I_{XY} & -I_{XZ}-\frac{P}{2}\ell^5 K_{\dot r} \\ mZ_G & 0 & -\frac{P}{2}\ell^4 M_{\dot w}-mX_G & -I_{XY} & I_Y-\frac{P}{2}\ell^5 M_{\dot q} & -I_{YZ} \\ -mY_G & -\frac{P}{2}\ell^4 N_{\dot v}+mX_G & 0 & -I_{XZ}-\frac{P}{2}\ell^5 N_{\dot p} & -I_{YZ} & I_Z-\frac{P}{2}\ell^5 N_{\dot r} \end{bmatrix}$$

The reason that $\dot x, \dot y, \dot z, \dot\theta, \dot\phi$ and $\dot\psi$ do not appear in the matrix is that they are not dependent on $\dot u$ etc., and therefore all of the cross terms are zero. This reduces the matrix to six levels instead of twelve. The solution to the equation is

$$[M][\dot X] = [X]$$

$$[M]^{-1}[M][\dot X] = [M^{-1}][X]$$

$$[I][\dot X] = [M^{-1}][X]$$

$$[\dot X] = [M]^{-1}[X]$$

where $[M]^{-1}$ is the inversion of the $[M]$ matrix. As long as the coefficients that make up the $[M]$ matrix do not change between each integration cycle, the matrix can be inverted once when the program is originally written and the inverted matrix used every time on the right hand side of the equation. Unfortunately, X_G, Y_G and Z_G are elements in this matrix and they represent the amount of water in the ballast tanks of a submarine. Therefore, it is necessary for an exact computation to calculate the elements of the matrix on each cycle. The matrix is then inverted with the aid of a subroutine. Each of the rates are calculated without the terms that are functions of the other rates. These particular rates are multiplied by the inverted matrix to give the true accelerations at that integration cycle time. The open integration method then integrates the rate to produce the next set of positions. The whole process is then repeated.

The submarine research program has a control card that can be used

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to remove the evaluation of the original matrix after the first cycle if none of the elements are going to change during the course of the program. This is the usual case if the tanks are not going to be changed.

Matrix inversion and multiplication take up both room and time in the computer program as anyone who has done matrix arithmetic knows. Therefore, a programing technique was borrowed from aircraft flight simulation to avoid this problem. From the NSRDC equations:

$\dot{u}$	requires	$\dot{r}$, $\dot{q}$
$\dot{v}$	requires	$\dot{p}$, $\dot{r}$
$\dot{w}$	requires	$\dot{q}$, $\dot{p}$
$\dot{p}$	requires	$\dot{r}$, $\dot{q}$, $\dot{w}$, $\dot{v}$
$\dot{q}$	requires	$\dot{p}$, $\dot{r}$, $\dot{u}$, $\dot{w}$
$\dot{r}$	requires	$\dot{q}$, $\dot{p}$, $\dot{v}$, $\dot{u}$

The usual case is for Y_G and the cross inertias to be zero due to symmetry of the submarine. If the matrix is not used,

$$\left. \begin{array}{l} \dot{u}_n \quad \text{will use } \dot{q}_{n-1} \\[2mm] \dot{v}_n \quad \text{will use } \dot{p}_{n-1}, \dot{r}_{n-1} \\[2mm] \dot{w}_n \quad \text{will use } \dot{q}_{n-1} \end{array} \right\} \text{then} \left\{ \begin{array}{l} \dot{p}_n \quad \text{will use } \dot{v}_n \\[2mm] \dot{q}_n \quad \text{will use } \dot{u}_n, \dot{w}_n \\[2mm] \dot{r}_n \quad \text{will use } \dot{u}_n \end{array} \right.$$

The other terms have zero for a coefficient. The effect of this procedure is a choice of either having the translational accelerations or the rotational accelerations slightly in error, depending on which set is calculated first.

The next parameter to be determined was the integration step size. This is a critical value because it determines the time available in real-time to compute the entire problem and service all I/O busses before the integration interval is used up. If this step size (the variable H in all computer programs) is too small, the computer will not finish the computations required in real-time. If H is too large, the accuracy suffers and finally, if H is increased beyond a certain limit, the integration method used will run into stability problems. Therefore, the trade-off is to make H as large as possible, consistent with both accuracy and stability.

The Runge-Kutta integration method supplied with the submarine research program has a feature that allows the step size to be adjusted between each cycle dependent on the accuracy required. Both the upper and lower bounds on H can be specified.

A typical program was run with the demonstration submarine in which the rudder was deflected to 35 degrees and sternplanes were deflected to fifteen degrees. A time interval of 200 seconds was considered sufficient to determine the effect of the variable step size. The maximum values of the twelve parameters were printed out and 0.1% of this value was used as the maximum allowable error for any step. The program was

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re-run with H allowed to vary between 0.05 and 2.0 seconds subject to the accuracy requirements. The run showed that the interval for 0.1% accuracy varied from 0.25 to 2 seconds. Many steps were one second or longer, indicating that the use of one second intervals is probably adequate for most maneuvers, but not necessarily good enough for disaster or emergency conditions. The smallest step size of 0.25 seconds occurred with step inputs and this indicated that this time interval was sufficient for any purpose. Therefore, the studies were run with an H of 0.25 seconds in all cases. However, any fixed step size can be chosen with any of the integration methods used in the subroutines.

An additional study was made into the largest possible step size based on a stability criterion that assumes the local growth of the propagated error must be kept within bounds. The criterion assumes that each integration method has a range that is a function of the step size and the partial differential of the function with respect to itself that will keep the propagated error bounded. The system may not be stable, even if this criterion is met, but it will certainly be unstable if it is not.

Assume that the function

$$\dot{y} = f(y, x_1, x_2, x_3, \text{ etc.})$$

is to be integrated by means of the 2nd order Adams method. The numerical quantity

$$h \cdot \frac{\partial f}{\partial y}$$

has to have a value between 0 and -1 for the integration to be stable. Looking at   for example,

$$\frac{\partial \ddot{w}}{\partial w} = \frac{\frac{p}{2} \ell^2 u}{m \frac{p}{2} \ell^3 Z_{\ddot{w}}} (Z_w + Z_{|w|}) \quad \text{where} \quad \begin{aligned} u &= \text{speed} \\ \ell &= \text{length} \\ m &= \text{mass} \\ p &= \text{density} \end{aligned}$$

If the specified values for the demonstration submarine are used, the value of the above function at high speed indicates that a step size of 2.1 seconds could be used. The other eleven integrations were checked and they give equally large maximum step sizes. The stability region for some common integration methods are listed in table 1. The theory for calculating the stability region for any method is given in Nigro[5].

The submarine research program has a control option that allows a program to be run either under the Runge-Kutta integration method, along with the matrix inversion and multiply, or to choose any one of a number of simpler methods in which the matrix is not used. Three are coded into the subroutine (INTEG); namely Euler, 2nd order Adams, and the best method based on stability alone. Table 25 in the programing report has 22

TABLE 1. INTEGRATION STABILITY REGIONS

Method	Stability Region
Euler	-2, 0
2nd Order Adams	-1, 0
Trapezoidal	- , 0
$O_{30}C_{31}$	-.82, 0
O_{33} Mod Gurk	-.82, 0
Best 2nd Order	-4, 0

different equations listed so that the subroutine can be recoded to utilize most of the popular numerical integration techniques. This makes it possible to have a whole library of integration methods and to use any one of them by means of a control card at the start of a run.

A number of the references express various integration methods as a Z-transform transfer function. The Z-transform is an extension of the traditional Laplace transform methods with the transform of a sampled function written in a simpler notation. The substitution $Z = e^{sT}$ is made where T is the sample step size and corresponds to the integration interval H. The Z-transform is a handy way of examining integration methods analytically because only knowledge of Laplace transforms is required to use them, but they can be confusing to program on a digital computer. Therefore, a method is given to convert any Z-transform transfer function into a difference equation. It can then be programed directly in FORTRAN.

The key to this procedure is that Z or e^{sT} is a shifting function in the time domain. Therefore, any negative power of Z times a parameter merely indicates that the value of the parameter so many cycles back from the present time should be used. An example will make this clear.

Baxter[6], for example, lists several integration methods I(Z). For the 2nd order Adams,

$$I(Z) = \left(\frac{h}{2}\right)\left(3Z^{-1} - Z^{-2}\right) / \left(1 - Z^{-1}\right)$$

The first step is to write I(Z) as a transfer function of Y(Z) to $\dot{Y}(Z)$.

$$\frac{Y(Z)}{\dot{Y}(Z)} = \frac{h}{2} \cdot \frac{3Z^{-1} - Z^{-2}}{1 - Z^{-1}}$$

Cross-multiply to clear the fraction

$$Y(z)(1-z^{-1}) = \dot{Y}(z) \cdot \frac{h}{2}(3z^{-1} - z^{-2})$$

$$Y(z) - Y(z)z^{-1} = \frac{h}{2}(3\dot{Y}(z)z^{-1} - \dot{Y}(z)z^{-2})$$

$$Y(z) = Y(z)z^{-1} + \frac{h}{2}(3\dot{Y}(z)z^{-1} - \dot{Y}(z)z^{-2})$$

This procedure assumes that the Z-transform is a linear operator which is to be expected since the Laplace transform is linear. The difference equation can now be written directly by making the substitution

$$Y(z) = Y_n, \quad Y(z)z^{-1} = Y_{n-1}, \quad etc.$$

which is the same difference equation stated earlier for 2nd order Adams. Any integration methods expressed as a Z-transform transfer function can be used in subroutine INTEG by coding a few lines of FORTRAN and recompiling this subroutine only.

A number of different runs were made which included both step inputs and realistic surface deflections. From these runs it is possible to draw some definite conclusions as to step size and integration methods to be used in a submarine simulator.

The Runge-Kutta method is more accurate than is necessary even for a research program. However, it serves as a useful check on other methods and, with a step size of 0.25 second, is the reference standard.

It is not necessary to use matrix multiplication to solve for simultaneous rates. Using the rates calculated in the last integration cycle does not affect the accuracy of the runs.

The minimum step size for best accuracy is 0.25 second based on the variable step size program. However, there is almost no difference when the program is run at 0.5 second. When it is run at 1.0 second, all parameters remain the same except for p. If the bank angle output is examined closely, it will be seen that the high frequency wiggles are out of phase with the standard and take a little longer to die out. At 1.5 seconds, p goes into oscillation. Therefore, a step size of 0.5 second should be used for a casualty trainer where high rates will be

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encountered. A step size of 1.0 second is satisfactory for a maneuvering trainer.

It was hoped that Euler's method could be used for the maneuvering trainer because it gave good results at 0.25 second except for p. However, at 1.0 second most of the parameters diverged, thus justifying its bad reputation. Therefore, 2nd order Adams will be used in the maneuvering trainer so that a large step size can be used.

C. COEFFICIENT REDUCTION

1. LONGITUDINAL IMPULSE TESTING

The equations of motion can be represented as a transfer function that takes a certain set of input conditions and transforms them into a certain set of outputs. The coefficients of the differential equations determine the characteristics of this transfer function, and as long as the system under consideration is linear, it is possible to express the response of the system independently of any set of inputs by means of the impulse function.

The impulse function is a spike that occurs at time "t=0" and shocks a system into oscillation. The system will oscillate because energy has been added. These oscillations will correspond to the natural frequencies and damping ratios because all inputs are zero immediately after "t=0". The time outputs can be analyzed to determine the coefficients necessary to describe a linear differential equation for the system. These will be related to the coefficients required in the equations of motion. The techniques for doing this are well known in Laplace transform theory applied to servo design[7].

A procedure somewhat analogous to impulse testing is used in the determination of handling characteristics during the full scale test of the actual submarine. The test is known as a "meander maneuver" and one can obtain a direct measure of the inherent longitudinal stability of the submarine with the following procedure. Essentially, the meander test is conducted by disturbing the submarine from its neutral pitch angle by deflecting the sternplanes a fixed amount in either rise or drive. After a predetermined pitch angle is reached, the planes are returned to their neutral position. The resulting pitch angle trajectory provides both visual and quantitative evaluations of the longitudinal stability of the submarine.

This comes as close as possible to impulse testing when applied to the real-world vehicle. Mathematically, an impulse occurs in zero time, but in the real-world case the surfaces take time to move and the controls have to be held in the deflected position so that the system is disturbed enough to give a good reading of pitch angle change. However, when the surface movement is plotted on the same time scale as the meander maneuver, it forms a very narrow pulse that approaches an impulse. Once the pitch angle has been plotted for any onset speed, it can be analyzed for such numerical maneuvers as damping ratio, undampened natural

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frequency, stability roots, and trajectory envelope decay. These are given in the evaluation reports of the handling qualities for each submarine class.

It was hoped that impulse testing of the equations of motion could serve two purposes: (1) A series of runs would be made to determine the natural frequencies and damping ratios at all speeds; and (2) these figures could then be compared with the actual figures on the submarine to check the accuracy of the complete set of coefficients. Once this had been done, the figures could be used to generate a new set in which the equations are assumed to be linear and therefore much simpler than the original set. The first purpose was successful, the second was not.

The following procedure is used to generate an impulse run. Computer program ZC790, Submarine Simulation, Longitudinal is used to generate a run with the control subroutine set to generate an input impulse and the output option set to punch cards with the time and pitch angle recorded every two seconds. The output on these cards is assumed to follow the form

$$Y = a_1 e^{-\delta t} + a_2 e^{\alpha t} (\cos \beta t + \psi)$$

where y and t are on the cards and $a_1, a_2, \delta, \alpha, \beta$, and ψ are constants that are to be determined.

The reason for choosing this form as an equation comes from the form of the characteristic equation for the longitude channel when the equations of motion are linearized. The Laplace transform of y is

$$\mathcal{L}[y(t)] = \frac{K_2 S^2 + K_1 S + K_0}{(S + \delta)((S + \alpha)^2 + \beta^2)}$$

α, β, and δ are the natural frequencies of the system and remain the same in either the time or complex domain and a_1, a_2, and ψ are functions of these and K_2, K_1, and K_0.

If we consider the equations of motion as a linearized set in the complex domain, they can be written as

$$[Y(S)] = [E(S)][C(S)]$$

where $Y(S)$ is the output matrix, $E(S)$ are the equations of motion and $C(S)$ are the input conditions. An impulse is a constant in the complex domain so $Y(S)$ is equal to $E(S)$. Therefore, $Y(t)$ in the time domain in response to an impulse can be determined by taking the inverse Laplace transform of the equations of motion for the particular input (the sternplanes in this case), and the particular output (pitch angle). The details of setting up the equation of motion for the longitudinal channel

are described in the programing report in program EC140 and will not be repeated here. From this analysis the characteristic equation is

$$A_3 S^3 + A_2 S^2 + A_1 S + A_0 = 0$$

where

$$A_3 = \left(\frac{P\ell^3}{2} Z_{\dot{w}} - m\right)\left(\frac{P\ell^5}{2} M_{\dot{q}} - I_y\right) - \left(\frac{P\ell^4}{2} M_{\dot{w}}\right)\left(\frac{P\ell^4}{2} Z_{\dot{q}}\right)$$

$$A_2 = \left(\frac{P\ell^2 u_o}{2} Z_w\right)\left(\frac{P\ell^5}{2} M_{\dot{q}}\right) + \left(\frac{P\ell^3}{2} Z_{\dot{w}} - m\right)\left(\frac{P\ell^4}{2} M_{\dot{q}}\right)$$
$$- \left(\frac{P\ell^4}{2} M_{\dot{w}}\right)\left(u_o m + \frac{P\ell^3}{2} u_o Z_q\right) - \left(\frac{P\ell^3}{2} u_o M_w\right)\left(\frac{P\ell^4}{2} Z_{\dot{q}}\right)$$

$$A_1 = \left(\frac{P\ell^2}{2} u_o Z_w\right)\left(\frac{P\ell^4}{2} u_o M_q\right) + \left(\frac{P\ell^3}{2} Z_{\dot{w}} - m\right)\left(B \cdot Z_B\right)$$
$$- \left(\frac{P\ell^3}{2} u_o M_w\right)\left(u_o m + \frac{P\ell^2}{2} u_o Z_q\right)$$

$$A_0 = \left(\frac{P\ell^2}{2} u_o Z_w\right)\left(B \cdot Z_B\right)$$

where all the symbols are defined in the equation of motion report. Actually, computer program EC140 solves this equation in direct matrix form and allows for the variation in speed.

A compensation is made for the non-linear terms in the longitudinal loop by modifying the basic coefficients. For example,

$$Z_{w|w|} \cdot W(w^2 + v^2)^{\frac{1}{2}} \quad \text{becomes} \quad Z_{w|w|} \overline{w} \cdot w$$

where $\overline{w}$ is an average value of w used as a constant. It is determined by running a number of full set runs and scanning the output for a typical value of w at steady state conditions. This allows ZW and ZWAW to be combined into a single term. A similar process is used with $|q|$ being set to $\overline{q}$.

It is obvious that the characteristic equation is a cubic with at least one real root. It can be assumed that the other two roots are a complex pair from the behavior of the submarine in a meander maneuver. Therefore, if the equations were linear, the frequencies could be determined directly from the coefficients and conversely, the coefficients could be calculated from the measured frequency response.

When the cards have been punched with the time and pitch angle; α, β, and γ can be determined by two other programs. EC310 is a program that performs a "least squares fit" to any specified equation

format from a series of X-Y points. This program requires that fairly close initial guesses be supplied for the parameters that are to be calculated. Program EC330 calculates a set of numerical values to be used as starting conditions for EC310. Details are left to the programing report.

A summary of the situation will be helpful at this point.

1. The handling qualities report for a particular submarine gives the results of meander tests taken during full-scale trials and expressed in terms of frequencies and delays.

2. The stability and control report gives the same information calculated from the coefficients as determined from the model tests.

3. Program EC140 uses the NSRDC coefficients to calculate the frequencies and damping ratios. The values are identical to the report if the coefficients given in the stability and control reports are used in this program.

4. Computer program ZC790 can be used to produce a time output which can be analyzed to find α, β, and γ. The natural frequency and damping ratio can then be calculated.

It would be convenient if all four sources agreed exactly, but they do not. Therefore, it was deemed advisable to determine the validity of the supplied coefficients and the programing of ZC790, the main simulation program.

The coefficients were taken as supplied by NSRDC and run in ZC790 to produce a full set of meander curves. These curves were compared with the new data acquired on the demonstration submarine and stored at NSRDC, and they matched over the entire speed range. Therefore, we can be assured that the longitudinal coefficients used and simulation program ZC790 (and EB920) will accurately simulate the demonstration submarine. The longitudinal and lateral channels of EB920 were checked out in another manner in order to give further support to this conclusion. The handling qualities report for the demonstration submarine gives a time history of a 20-knot vertical-plane overshoot maneuver and a time history of a typical 24-knot submerged turn. The input conditions were duplicated as closely as possible and the output curves matched the actual output data taken during sea-trials to a much closer degree than required for an accurate simulator.

We can now establish a data platform from the approved criteria for any submarine simulator. The coefficients as received from NSRDC are inserted into ZC790 for longitudinal tests or EC920 for six-degrees-of-freedom and any specified set of inputs can be run off. This will not only serve to check the reduction made as a part of this study but more importantly will serve to check the operation of any simulator using the standard set of equations of motion. Even before a simulator is delivered to the user, a complete set of performance specifications can be computed.

An investigation was made into why the handling qualities report and the stability and control report should have slightly different frequency and damping ratio parameters for what is essentially the same vehicle. It was concluded that the processing of the data was responsible for this difference. The frequencies in the stability report are calculated from the coefficients as determined by model tests using the linear model described in EC140. This neglects such coefficients as $M_{\dot w}$ w which were proven to be important by the investigation. The frequencies in the handling qualities report are determined from a time-history on the assumption that a single pair of complex poles are present to represent the system rather than a real pole and a pair of complex ones. This is a standard treatment in servo texts and is used to calculate damping ratios in the verification section of this study. It can lead to different numbers depending on which points of the time-history are used. Authorities are not in agreement as to the way to measure the damping ratio of a higher order function, and most of them use only the single complex pair case as an example.

The method of calculating the frequencies from the linear coefficients is possible lead to an idea for calculating a simplified set of longitudinal coefficients that seemed extremely attractive in theory. The full set of coefficients are used to run off a set of impulse curves at all speeds. These curves are reduced to numerical values by the technique described, and the reverse equations contained in EC140 are used to generate a new set of coefficients for all speeds. These would be linear, but would contain all the variations described by the complete set because of this change with speed. Unfortunately, this attempt proved to be unsuccessful due to the difficulty of solving the reverse set on a "least squares" fit. The approach is outlined here because the problem is in the computing methods available and not in the theoretical basis behind the problem.

The unknowns are the eight basic coefficients:

$$Z_w, Z_q, M_w, M_q, Z_{\dot w}, Z_{\dot q}, M_{\dot w}, M_{\dot q}$$

which are used to form the characteristic equation:

$$A_3 S^3 + A_2 S^2 + A_1 S + A_0 = 0$$

If we assume that the variation of the longitudinal stability indices with speed are known, (these are the α, β and γ's from either the impulse runs or the stability and control report) the characteristic equation takes the form

$$(S + \gamma)((S + \alpha)^2 + \beta^2) = 0$$

Therefore, we can set up a one-to-one correspondence with the stability indices and the A's,

$$A_2/A_3 = 2\alpha + \gamma$$

$$A_1/A_3 = 2\gamma\alpha + \alpha^2 + \beta$$

$$A_0/A_3 = \gamma(\alpha^2 + \beta^2)$$

and then solve for the basic coefficients. If tests are run at more than three speeds, more knowns than unknowns are generated and a best fit of runs at one-knot intervals, for example, would give us a basic set that could be used to linearize its equation of motion.

A number of different numerical methods were tried and discarded because they all broke down on the fact that the slopes of the change of coefficients with respect to speed is so shallow that not even double precision subroutines were able to prevent the problem from blowing up. Most of the non-linear equation solvers require an initial guess and the shallower the slope, the closer the guess has to be to the true answer for all eight coefficients. Usually, the original values are suitable as initial guesses, but in this case they do not satisfy the requirements. The mathematical difficulties are covered in detail in Freeburg[8]. Nevertheless, this approach might prove useful in further work in this field.

2. BASIC COEFFICIENT ANALYSIS

The equations of motion of a body can be developed in terms of force equations derived from d'Alembert's principle. This principle states that for any body in motion, the applied forces must be in equilibrium with the inertia reactions of the body. By assuming that the inertia reactions have the dimensions of force, the dynamic problem can be reduced to a static one in which the only requirement is that of "sum of forces = 0". Once the force equations are developed and expressed in terms of coefficients times functions of velocities, angles, etc., the accelerations can be computed. Known velocities with the external forces (rudder angle, plane angles, and thrust) are used as input conditions. This assumes that the mass and inertias are known. The accelerations are then integrated to give a new set of velocities which are then used to recompute a new set of accelerations.

The coefficients can be divided into three categories to facilitate the investigation of their effects on the acceleration computation for any set of inputs. They are longitudinal, lateral, and cross-coupling coefficients.

In the general case, there are six simultaneous differential equations to be solved for the six-degrees-of-freedom for a body that can rotate

and translate. However, if certain conditions are placed on the inputs,
the problem can be greatly simplified. It is helpful to start the inte-
gration with all forces exactly balanced by the control surface deflec-
tions so that the six accelerations are zero. This is fairly simple to
achieve in a computer program by setting the steady-state inputs and
velocities to the correct precomputed values. When the program begins
to run, the accelerations are zero and the velocities do not change until
a surface is deflected.

The first set to be investigated are the longitudinal coefficients.
The program starts with the rudder at neutral and the lateral axis veloc-
ities at zero. Since the longitudinal axis velocities do not couple into
the lateral channel, the lateral velocities remain at zero and any coef-
ficients multiplied by these velocities can be ignored if only the diving
planes and thrust are varied. The three-degree-of-freedom research
program not only assumes that this condition holds, but it eliminates
the lateral channel entirely along with any coefficients using lateral
velocities in the longitudinal channel.

Therefore, only those coefficients multiplied by the longitudinal
velocities are of interest in the first category. They are listed in
table 2.

TABLE 2. LONGITUDINAL COEFFICIENTS

Axial Force	Normal Force	Pitching Moment
XQQ	ZDQ	MQD
XUD	ZWD	MQAQ
XWQ	ZQ	MWD
XUU	ZAQDS	MQ
XWW.	ZWAQ	MAQDS
XDSDS	ZSTR	MAWQ
XDBDB	ZW	MSTR
Thrust coefficients	ZWAW	MW
XWWE	ZAW	MWAW
XDSDSE	ZWW	MAW
	ZDS	MWW
	ZDB	MDS
	ZQE	MDB
	ZWE	MQE
	ZWAWE	MWE
	ZDSE	MWAWE
		MDSE

The second set of coefficients are those that govern the behavior
of the lateral channel. The assumption is made that an autopilot is
attempting to keep the submarine level so that q is zero and the value
of w remains a small constant. There will be some coupling through the
change of forward speed as the vehicle begins to turn, but it is assumed
that this is small compared to the effect of the rudder. The lateral
coefficients are given in table 3.

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TABLE 3. LATERAL COEFFICIENTS

Lateral Force	Rolling Moment	Yawing Moment
YRD	KPD	NRD
YPD	KRD	NPD
YPAP	KPAP	NRAR
YVD	KP	NVD
YR	KR	NP
YP	KVD	NR
YARDR	KSTR	NARDR
YVAR	KV	NAVR
YSTR	KVAV	NSTR
YV	KDR	NV
YVAV	KSTRE	NVAV
YDR		NDR
YRE		NRE
YVE		NVE
YVAVE		NVAVE
YDRE		NDRE

Finally, there are the cross-coupling terms that couple one channel into the other and only come into prominence during a climbing or diving turn. The cross-coupling terms are given in table 4.

TABLE 4. CROSS-COUPLING TERMS

Axial	Lateral	Normal	Rolling	Pitching	Yawing
XRR	YPQ	ZPP	KQR	MPP	NPQ
XRP	YQR	ZRR	KPQ	MRR	NQR
XVR	YVQ	ZRP	KVQ	MRP	NWR
XVV	YWP	ZVR	KWP	MVR	NWP
XDRDR	YWR	ZVP	KWR	MVP	NVQ
XVVE	YVW	ZVV	KVW	MVV	NVW
XDRDRE					

It should be noticed that the cross-coupling terms in the lateral channels always have one lateral velocity as a multipling factor, but the longitudinal terms are always multiplied by two lateral velocities. This is why longitudinal variations do not affect the lateral channel when the lateral velocities are zero, but any turning rate will affect the performance of the longitudinal channel.

It was established above that the NSRDC equations are accurate for the simulation of operational submarine motion when they are used with the full set of coefficients supplied by this facility. Therefore, a certain amount of program complexity has already been eliminated during the analysis performed at NSRDC. The terms which are omitted can be of three types. The first type are those for which the coefficient is zero because of geometrical symmetry of the hull or mathematical symmetry of the equations of the hydrodynamic reactions in Taylor's theorem expansion.

This category includes the following:

XUU	YQR	ZWW	KPQ	MQAQ	NSTR
	YSTR	ZAW	KSTR	MAW	
	YPAP		KVW	MWW	

The second type are those which can exist but have been determined to be too small for submarines in general. They include:

YVQ	ZPP	KDR	MPP	NQR
YWR		KR		NRAR
		KVQ		NWR
		KWR		

Finally, there are those coefficients which have been set to zero because they have not been investigated and the effect of their presence and their magnitude is unknown.

The η terms are included in this category. They cover the effect of the propeller disturbing the flow in addition to producing thrust in the body of the submarine. As long as a propeller is subjected to uniform axial flow, the stream velocity is constant for all blade positions around its path of rotation. As a result, the angle of attack of the blade, the thrust, and torque are constant, and the η coefficients can be set to zero.

As a rule, stern propellers must operate in non-axial flow conditions. This is true even for an axially mounted propeller on a body of revolution where the flow is symmetrical and converges around the afterbody. Then too, the body axis may not be parallel to the direction of the stream velocity, resulting in further deflection of the flow. The blade angle of attack is altered with respect to flow direction, thus a greater thrust is produced at one side resulting in an upward force being applied to the shaft. Any change in the ship's direction of motion will be reflected by the hydrodynamic forces and moments. A change in the propeller RPM will result in a changed force diagram. The propeller blades will be acting at different angles of attack with respect to the fluid velocity vectors until steady-state conditions again prevail after the propeller velocity is changed.

The demonstration submarine is known to have certain non-linear effects that are not reflected in the equations of motion with the present set of coefficients. These effects could be provided through the addition of the η terms, but their magnitudes are not known at the present time. However, the NSRDC equations of motion report states that for the moderate changes in ahead speed involved in most normal maneuvers, all of the $(\eta -1)$ terms usually can be neglected. Therefore, the following coefficients are set to zero under the assumption that axial flow exists at the propeller at all times.

XVVE	YRE	ZQE	KSTRE	MQE	NRE
XWWE	YVE	ZWE		MWE	NVE
XDRDRE	YVAVE	ZWAWE		MWAWE	NVAVE
XDSDSE	YDRE	ZDSE		MDSE	NDRE

However, all of these terms are included in the programing so that if the necessity of using them should arise, they can be added merely by placing a number on an input card.

A survey was made of four different submarine simulations, and in each case the coefficients listed above were not used. This survey was correlated with interviews of personnel who had received training in the simulators in question and had served on the submarine class being simulated. The conclusion reached was that the simulation was a good representation under normal training maneuvers even with these terms missing.

3. INERTIAL TERMS

It was stated that the equations of motion can be derived from d'Alembert's principle that the applied forces must be in equilibrium with the inertial reactions of the body. This section examines these inertial reactions of the body with respect to submarine simulation.

The inertial terms are those terms that result from the mass of the vehicle rather than from any outside forces on the vehicle. As such, they are written on the left hand side of the equation in the standard equations of motion published by NSRDC. These terms are of two types; those resulting from mass and inertia of the hull, and those that appear in the equations because the center of gravity does not remain at the center of the coordinate system chosen to specify the coefficients.

Generally, the coordinate system used in aircraft simulation is coincidental with the center of gravity and moves along with it. This change does not affect the coefficient because the shift is small and is mainly due to depletion of on-board fuel located in tanks close to the CG.

An entirely different case is found in submarine simulation. Large weights of water are taken on board in tanks far removed from the origin of the reference point used for the inertial frame. Therefore, the parameters X_G, Y_G, and Z_G representing the shift of the CG off the origin are included in the simulation.

The first step was to eliminate functions that are zero because of the nature of the shape of a submarine. It can be assumed that Y_G can be set to zero because the CG is not going to shift far in the lateral direction as the ballast tanks are filled. These tanks are symmetrical with reference to the X-Z plane. I_y has the same value as I_z and the cross inertial I_{xy}, I_{xz}, and I_{yz} are zero on all submarines studied. This reduces the inertial terms of the equations to:

Axial
$$-m\left(\dot{u}-vr+wq-X_G\left(q^2+r^2\right)+Z_G\left(pr+\dot{q}\right)\right)$$

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Lateral $\quad - m\left(\dot{v} - wp + ur + Z_G(qr - \dot{p}) + X_G(qp + \dot{r})\right)$

Normal $\quad - m\left(\dot{w} - uq + vp - Z_G(p^2 + r^2) + X_G(rp + \dot{q})\right)$

Roll $\quad - I_x \dot{p} - mZ_G(\dot{v} - wp + ur)$

Pitch $\quad - I_y \dot{q} + (I_x - I_z)rp + m\left(Z_G(\dot{u} - vr + wq) - X_G(\dot{w} - uq + vp)\right)$

Yaw $\quad - I_z \dot{r} + (I_y - I_x)pq + mX_G(\dot{v} - wp + ur)$

The various values of cross velocities were kept in the reduced equations of motion without investigating their magnitude of effect. This is because all of these terms are needed to calculate the forces from the coefficients, and therefore they can be included at the cost of a single addition. Also, any one of these terms can become quite large under casualty conditions and will therefore be needed for good recovery simulation.

If X_G and Z_G can be held at zero, as in an aircraft simulation, the inertial terms could be greatly reduced in complexity. Therefore, an investigation was made into the effect of X_G and Z_G on a typical simulation.

The first task is to determine the range of X_G and Z_G. The equations for these parameters are

$$X_G = \frac{W_o X_o + \sum_1^N W_{Ti} X_i}{W}$$

$$Z_G = \frac{W_o Z_o + \sum_1^N W_{Ti} Z_i}{W}$$

$$W = W_o + \sum_i^N W_{Ti}$$

where the W's are the weights of the submarine and water in the tanks and the Z's and X's are the coordinates of the respective water ballast tanks. The figures used for the investigation are contained in the report "Measures of Performance Based on Motion Simulation" for the demonstration submarine. In the case of flooding, a compartment filled with water can be treated as an additional tank.

Program EC430 calculated X_G, Y_G, Z_G, and W for any tank arrangement. The data on the submarine and the individual tanks is used as an input, and any tank arrangement is controlled by an input card following the data. In one of the submarines studies, there were values of Y_G associated with individual tanks, but the shift in Y_G was very small. Also, where tanks are split laterally, the two sides are always filled equally, reducing Y_G to zero.

A number of different tank arrangements were tried to cover a range of X_G and Z_G. The final values used were Z_G closest to the center of buoyancy, a normal operating condition (W=B and equal distribution in the trim tanks), and a worst case in X_G with major flooding at the forward end of the submarine. Tests were made at 5 and 25 knots to cover the operating range under these conditions.

A normal overshoot maneuver is used to check the effects of varying X_G and Z_G. Programs EB920 or ZC790 are used with the different values and u, theta, and z are recorded. The results are given in figure 4.

1 - Ref., $X_G = Z_G = 0$
2 - Z_G Maximum
3 - Normal
4 - X_G Maximum
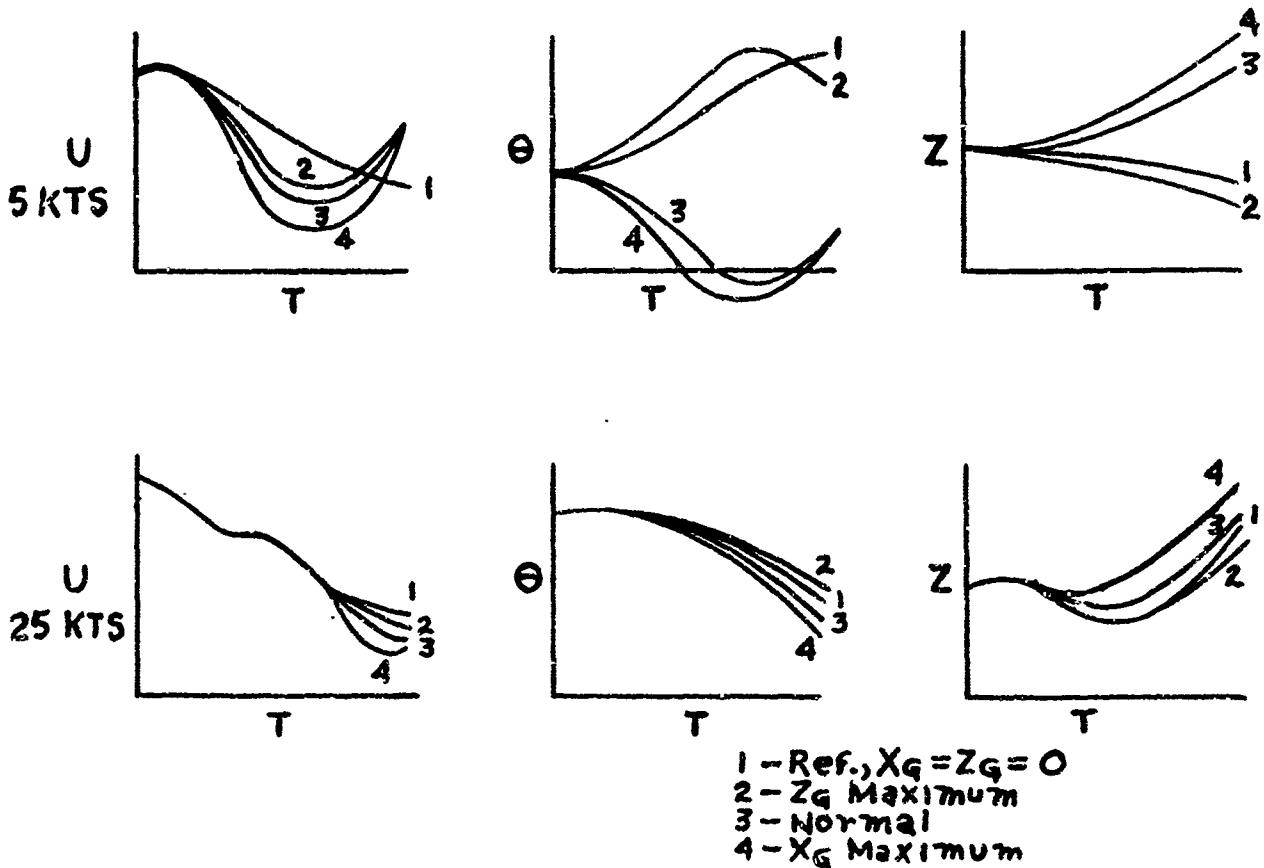
Figure 4. Overshoot Response to Tankage Variation

It should be noted that these curves do not represent the true response of the submarine from the change in the center of gravity due to distribution of ballast. This is because only X_G and Z_G is changed. The weight still equals the buoyancy.

It can be seen that at 25 knots there is very little change in response to changing the two parameters. However, at 5 knots, the change is drastic; in fact, the pitch angle effect reverses with the change in X_G. The change in Z_G is not as great.

Program ZC691 was used to find the amount of error in three cases.

TABLE 5. CHANGE OF PERFORMANCE WITH TANKAGE CHANGE

| | PERCENT CHANGE OF VARIABLE | | |
	Δu	$\Delta \theta$	Δz
		5 KNOTS	
Z_G MAXIMUM	-1.3	-6.6	-23.4
NORMAL	-372.6	-681.2	-1307.7
X_G MAXIMUM	-692.1	-856.8	-1545.0
		25 KNOTS	
Z_G MAXIMUM	-2.9	0.2	7.7
NORMAL	-11.6	-16.4	-41.4
X_G MAXIMUM	-16.0	-20.7	-53.5

Z_G could be set to zero if it were not for the 23.4% change in maximum depth at 5 knots. The direction of change of z remains the same.

A consideration of the results leads to the conclusion that X_G and Z_G should be kept in the modified equations of motion set, and Z_G can be set to a constant in the basic set.

The degree of variation in X_G and Z_G can be used to determine how accurate the tank representation should be in a submarine casualty control trainer. The parameters were varied and changes measured at the end of one minute. A 15% change in Z_G resulted in no change in u, less than 1% change in the response of theta, and about a 2% change in maximum depth change. A 5% change in X_G resulted in as great a change in u, and a 3% change in theta. Z was changed very little. Therefore, a very close simulation of Z_G in the tank representation is unnecessary even in a casualty trainer, but the factors that go to make up X_G should be computed to within 5% of the actual response in both time and weight.

The standard equations of motion allow for the center of buoyancy (CB) to be displaced from the center of the coordinate axis as well as the center of gravity. These parameters are expressed as functions of X_B, Y_B, Z_B, which are the distances in each dimension of the displacement. The CB is fixed in the submarine, and the coefficient coordinate system is always chosen so that X_B and Y_B are zero. Z_B has to have a definite value that puts the CB above the CG when the submarine is submerged. This causes the mass to act as a stable pendulum which returns to neutral at the natural frequency of the system when the outside disturbing forces are removed. The restoring force is the force of gravity and the equations of motion become

Axial $\quad -(W-B) \sin \theta$

Lateral $\quad (W-B) \cos \theta \sin \phi$

Normal $\quad (W-B) \cos \theta \cos \phi$

Roll $-(Z_G W - Z_B B) \cos \Theta \sin \Phi$

Pitch $-X_G W \cos \Theta \cos \Phi - (Z_G W - Z_B B) \sin \Theta$

Yaw $X_G W \cos \Theta \sin \Phi$

with the X_B, Y_B, and Y_G set to zero. It can be seen that time will be saved if terms such as "$\cos \Theta \sin \Phi$" are computed only once and saved for use in the valuation of the accelerations. This also applies to "$Z_G W - Z_B B$". The multiplication does not have to be made when separate tanks are being simulated.

$$Z_G W = \frac{W_o X_o + \Sigma W_{Ti} \cdot X_{Ti}}{W} \cdot W$$

$$= W_o X_o + \Sigma W_{Ti} \cdot X_i$$

and the latter term can be used before the division by W is made.

4. LONGITUDINAL COEFFICIENT REDUCTION

Although different sets of operations were followed for each group of coefficients, a general reduction procedure was followed that was common to all groups. The task was broken up into three parts: first, an analysis to determine the effect of each coefficient on the quality of simulation; second, the determination of the effect of removing groups of coefficients; and finally, the degree to which the remaining coefficients should be changed in order to improve the output response of the remaining set of coefficients after the reduction is made.

A data platform is established by treating a set of input conditions that will vary those parameters that are multiplied by the coefficients of interest. The full set of coefficients is used to run a program over a sufficiently long period of time to set the trend of the outputs to this set of input conditions. Points of interest occur where pitch changes direction or where most of the variables have reached a steady-state condition. Most of these runs are about one or two minutes simulated time in length. The variables used in the verification runs should be those which the trainee will observe during trainer operation such as pitch angle, bank angle, and forward velocity.

The simulation program is run a number of times with the same input conditions and the coefficients are set to zero on a term by term basis. The percentage change is calculated at the point of interest and a table is set up so that the effect of removing any one coefficient can be seen at a glance.

Some of the coefficients are not tested because the nature of the program requires a numerical value or else the output will not resemble any vehicle, much less a submarine.

The program is rerun with all coefficients having small percentage errors set to zero. The usual effect is to have all errors increased by

more than the amount of the separate runs added together because of the interactions contained in the program. Terms are added until an acceptable level of error is reached or remaining terms are used to compensate for the changes.

Compensation by adjusting the remaining coefficients seems attractive at first glance, but it cannot be carried too far. This is logically obvious because of the amount of work that has been done on the original equations of motion. Any term that could be combined with another term over the full range of operation of the vehicle would have already been combined with that term when the coefficients were supplied by NSRDC. However, the requirements for a training simulator are not as rigorous as those needed for research, so simplifications are possible.

In the sections that follow, the coefficients will be divided into natural categories and the method taken to reach a successful conclusion will be outlined. What cannot be shown in a report is that coefficient reduction is an art rather than a science. The basic assumptions and conclusions will hold true for any submarine chosen for simulation, but a cut and try procedure will be necessary before it is proven that the exact reduction used is effective in any particular case.

The longitudinal coefficients can be divided into two classes after the coefficients that have been previously set to zero are eliminated. They are those which are necessary for the simulation to operate and those which are to be investigated. The two classes are given in table 6.

TABLE 6. NON-ZERO LONGITUDINAL COEFFICIENTS

	Axial	Normal	Pitch
Necessary	XUD XDSDS XDBDB	ZQD ZWD ZQ ZSTR ZW ZDS ZDB	MQD MWD MQ MSTR MW MDS MDB
To be investigated	XQQ XWQ XWW	ZAQDS ZWAQ ZWAW	MAQDS MAWQ MWAW

The velocity and acceleration coefficients are required to give stability to the differential equation solutions. ZSTR and MSTR are required to give correct solutions to steady-state conditions when the submarine is in level flight. The effect of eliminating them is discussed later. The rest are required so that operation of the planes will result in response of vehicle movement in the proper direction.

This does not seem to leave many coefficients, but if the remaining ones are examined it will be seen that eliminating them will reduce the

complexity of the equation to a much greater degree than their number would indicate. Let us look at the ZWAQ as a horrible example. It is multiplied by $W/|W| \cdot |(V^2 + W^2)^{\frac{1}{2}}| \cdot |q|$ which requires five multiplications, a division, a square root, and three absolute values.

A set of initial conditions has to be established before investigation of the longitudinal loop can begin. The natural set would be to establish level flight before each run. This is not as easy as it sounds, and a side investigation was made into the subject of critical speeds and neutral angles for a submarine. Neutral angles are defined as those steady pitch angles, sailplane angles, and sternplane angles which, when combined, produce a condition of steady level flight or zero path angle at a particular speed. This requires that the angle of attack equal the pitch angle, and that the accelerations $\dot{W}$ and $\dot{q}$ equal zero. Reference is made to angle of attack in many texts, including the NSRDC reports, but the parameter does not appear in the equations of motion. This is because a body axis system of coordinates is used instead of a flight path set. The angle of attack is the angle between the flight path and the body when roll angle is zero.
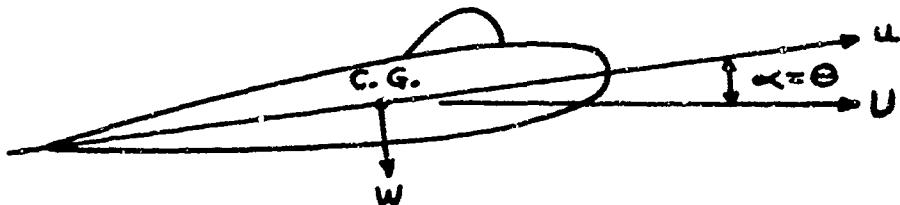
Figure 5. Angle of Attack

The angle of attack, and in the case of steady level flight the pitch angle, can be calculated from the relationship

$$U^2 = u^2 + w^2$$

$$\sin\Theta = \sin\alpha = \frac{w}{U} = \frac{w}{\sqrt{u^2 + w^2}}$$

as seen in figure 5. This removes the parameter Θ by replacing it with a function of w. For any speed u, the equations of motion can be solved for level flight in terms of w and δ_s or δ_b by setting $\dot{w}$, $\dot{q}$, and q to zero.

From Normal Equation (3)

$$Z_{w|w|} \cdot W|W| + Z_{ww} \cdot W^2 + Z_w \cdot u \cdot w + Z_{|w|} \cdot u \cdot |w| + Z_* \cdot u^2 + Z_\delta \cdot u^2 \cdot \delta = 0$$

From Pitch Equation (5)

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$$M_{w|w|} \cdot W|W| + M_{ww} \cdot W^2 + M_w \cdot u \cdot w + M_{|w|} \cdot u \cdot |w| + M_* \cdot u^2 + M_\delta \cdot u^2 \delta$$

$$+ \frac{2B \cdot Z_B \cdot w}{\rho \ell^3 \sqrt{w^2 + u^2}} = 0$$

ρ = density of water, 1 = length of submarine

δ can be either δ_b or δ_s provided that the correct coefficients $Z_{\delta b}, M_{\delta b}$ or $Z_{\delta s}, M_{\delta s}$ are used as inputs.

Program EC470 was written to solve the equations for w and δ. This program utilizes all coefficients of interest, so that it can be used with any set of coefficients applicable to the NSRDC equations. The pitch equation is solved for δ with a trial value of w, and this value is used in the normal equation via Newton's method. Once w and δ have been calculated, the program calculates

$$\Theta = \tan^{-1} \frac{w}{u} \qquad \text{radians}$$

$$\Theta_d = 57.3 \, \Theta \qquad \text{degrees}$$

$$\delta_d = 57.3 \, \delta \qquad \text{degrees}$$

and prints out u in ft./sec. and knots as well.

This program serves another purpose in addition to determining the initial conditions for longitudinal runs. The values for δ are determined for the operational speed range of the submarine. At some point, the values become very large. This speed is known as the critical speed because at this point the controls are ineffective in controlling the pitch attitude of the submarine. Fortunately, the critical speed is different for the sternplanes and the sailplanes. The sailplanes are effective at low speeds, below ten knots. The sternplanes have a very interesting effect that must be simulated correctly in order not to introduce negative training into the simulator. At a certain low speed, the neutral angles suddenly become very large. Pitch angle and sternplane deflection reverse direction over a very narrow range. Just below this speed, the angles come back to approximately their former position. The "Model Investigation of the Stability and Control Characteristics" report for the demonstration submarine has a graph of this behavior. It was used to check the steady-state angles generated by the simulation program when using the original coefficients. There was a perfect fit between program EC470 output and the NSRDC data for both δ_b and δ_s over the operational range of the demonstration submarine.

The low speed critical area is vital to effective submarine training. Therefore, if any coefficient changes affect this determination to any great degree, they must keep their original value in the modified simulation. The basic simulation is not as critical since a particular submarine is not being simulated. This is the reason for keeping ZSTR and MSTR on the list of necessary coefficients. ZWA. .nd MWAW should be checked closely because they have a definite effect on the values produced by EC470. ZWW, ZAW, MWW, and MAW have already been set to zero in the original analysis by NSRDC.

It was hoped that a new set of linear coefficients could be generated by means of impulse testing, but this did not work in the case of the longitudinal loop. Therefore, a series of overshoot runs were made to determine the effect of setting all coefficients in the second class to zero. This input was the standard overshoot schedule.
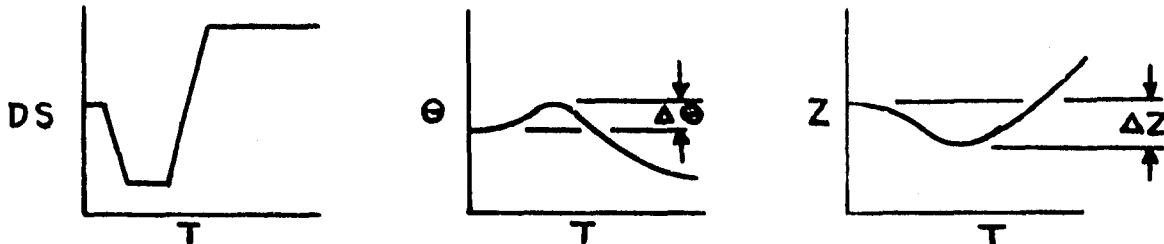
Figure 6. Comparison Criteria

The comparison procedure used for the longitudinal channel uses the maximum change in pitch angle and depth as the reference points. In order to remove any bias due to the particular overshoot maneuver used, all percentages are calculated from change of the change at the reference. If $\Delta\Theta$ -full set and $\Delta\Theta$ -modified set are compared, the percentage change is

$$\% \Delta\Theta = \frac{\Delta\Theta_{FS} - \Delta\Theta_m}{\Delta\Theta_{FS}} \times 100$$

The same holds true for changes in z. Program simulation times are set to be large enough so that the reference points are always reached for all runs as shown in figure 6.

A large number of runs were made. Table 5 gives several representative cases.

ZWAQ is an extremely desirable term to remove, and it seems to compensate nicely for MAQDS and ZAQDS. Therefore, these are set to zero in both programs.

In view of the fact that such a large change occurs with the deletion

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TABLE 7. PERCENT CHANGE IN OVERSHOOT, COEFFICIENTS SET TO ZERO

Coefficient = 0	%	%
ZAQDS	2.3	0.4
ZWAQ	2.3	0.0
ZWAW	4.6	0.1
MAQDS	-2.3	-1.8
MAWQ	4.6	0.3
MWAW	7.0	3.3
ZAQDS		
ZWAQ = 0	0.0	0.4
MAQDS		
All zero	20.9	13.6
All zero		
ZW + 75%	0	0.2
MW - 11%		

of ZWAW, MAWQ, and MWAW and that ZWAW and MWAW are two of the coefficients that determine the critical speed, it was decided that these three would be kept in the modified set of equations. This does not increase the amount of computation by a large amount once the factor $(v^2 + w^2)^{1/2}$ has been calculated. Since it was found necessary to calculate the square root for another reason, the most these three terms will add is six multiplications, three additions, and three data storage locations for the coefficients.

There is another extremely important reason for keeping ZWAW, MWAW, and MAWQ in the modified simulation. These terms govern the action of the submarine if it is going straight up. This would not happen under ordinary maneuvering procedures, but during casualty conditions it is possible to blow the main ballast tank while sitting dead in the water. This causes an upward movement in which w is the only body velocity to have any magnitude. The conditions are equivalent to an angle of attack of ninety degrees at this time.

The impulse runs did show that it is possible to produce the same output with a linear set of coefficients even if they could not be determined. Therefore, all the terms were set to zero in spite of the large errors, and the remaining coefficients were varied on a cut and try basis to see if the errors could be reduced. Runs showed that the acceleration coefficients ZWD, ZQD, etc. had very little effect on the magnitude of pitch angle and depth and only varied the frequency response. MQ and ZQ were a little more sensitive and the greatest response was to changing MW and ZW. Seven runs determined that ZW should be increased by 75% and MW was decreased by 11%. The magnitude of the response was excellent and the timing was not greatly altered. Additional runs showed that XQQ, XWQ, and XWW did not affect either longitudinal loop or the acceleration-deacceleration response when thrust was changed. Therefore, these coefficients were also set to zero.

5. LATERAL COEFFICIENT REDUCTION

The lateral coefficients can be divided in the same manner as the longitudinal coefficients.

TABLE 8. NON-ZERO LATERAL COEFFICIENTS

	Lateral	Roll	Yaw
Necessary	YRD YPD YVD YR YP YV YDR	KRD KPD KVD KP KV	NRD NPD NVD NR NP NV NDR
To be investigated	YARDR YVAR YVAV	KPAP KVAV	NARDR NAVR NVAV

Again, the number of coefficients that can be eliminated seems to be small, but removing YVAR would help in particular with simplifying the program.

The lateral loop is more complex than the longitudinal loop because three directions are interacting rather than just two. In spite of this, impulse testing provided information of value unlike the other case.

The procedure is as follows. A run is made using the full simulation program EB920. A new set of linear coefficients is generated from the assumption that the non-linear terms can be replaced by adjustments in the remaining coefficients. The EB920 run, made under autopilot control with a rudder impulse at t=0, is used to determine an average constant value of v and r under the assumption that w is zero. These values are used in program EC320, lateral characteristic equation calculator, to solve for the adjusted coefficients and natural frequencies using the principles of linear servo theory. The process is identical to that explained in the section on longitudinal impulse testing, except that the lateral characteristics equation has four roots instead of three. Two of the roots are real so that time response to an impulse becomes

$$\phi(t) = a_1 e^{-\alpha t} + a_2 e^{-\alpha t} \cos(\beta t + \psi) + a_3 e^{-\delta t}$$

The additional term over the longitudinal case is a very slowly converging spiral mode that is present in almost all six-degree-of-freedom vehicles. Pilots are generally unaware that this mode exists, because they automatically correct bank angle for it while piloting the vehicle. In aircraft, this mode often diverges slowly and will cause

entrance to a spin if the pilot's mind wanders for any length of time.

Cards are punched with the values of bank angle and time and a convergence run is produced so the actual frequency, rather than the estimated one, can be used to plot the output. The values of estimated frequencies produced by EC320 along with the cards are used by program EC120, time response coefficient estimator, to calculate input values for Brown's convergence routine (program EC310) for eight variables instead of six. EC120 will take α, β, γ, and δ calculated by EC320 and output a_1, a_2, a_3, and ψ as defined by the time response equation. Generally, Brown's routine is unreliable when eight variables are to be optimized so the estimated values for a_3 and δ are held constant for the first convergence run while optimizing the other six.

EC320 also produces adjusted values for YV, YP, KV, NN, and NR. These values are used in the complete simulation program with YARDF, YVAR, YVAV, KPAP, KVAV, NARDR, NAVR, and NVAV set to zero. Impulse runs were made at 5, 10, and 25 knots, and the results were plotted against the full-set runs determined above. The curves show excellent agreement between the two impulse runs at 5 and 10 knots and good agreement at 25 knots. This states that the non-linear coefficients can be set to zero and the lateral loop will still have the same natural frequencies and damping characteristics as with the full set of coefficients. Therefore, it was assumed that this procedure would lead to the basic set.

It was discovered later on that this approximation was too drastic when all the modifications were made to the modified and basic programs. Additional tests were made under closed loop control to simulate the response when the pilot controls the vehicle.

The trouble with making this type of investigation on a general purpose digital computer is that it is not a simulator. There is no way to have a man monitor the output and change δ_s, for example, as the program proceeds. A set of input conditions is established and the program calculates to the end of a set time. In order to simulate closed loop response, an autopilot was designed to keep the submarine in level flight at a constant depth. Otherwise, the longitudinal loop was free to move to any position during the course of a steady-state turn. The equation used for the autopilot was

$$DS = DB = .008(ZC-Z) + 3.5\theta + .012(u \sin\theta - w \cos\theta) + 2.0q$$

DS and DB are the stern and sailplanes respectively in radians, ZC is the command depth in feet, and the other variables are the same as in the simulation program. The term $(u \sin\theta - w \cos\theta)$ was used for $\dot{z}$ because $\dot{z}$ was not available in the CONTROL subroutine. The autopilot programing limited both plane deflections to $35°$ in either direction.

Closed loop control was found to be extremely necessary to determine that approximations made to reduce the coefficients were still valid

when the simulation was tested. For example, when the longitudinal loop was frozen ($w = 0$, $\dot{\theta} = 0$) the lateral loop responded very closely to the full set of coefficients to constant rudder commands when the non-linear terms were eliminated. This was not the case when closed loop control was used. Therefore, all tests were made with both the lateral and cross-coupling terms using the autopilot with a steady turn requirement. Even though the submarine was being held in level flight, the behavior of the autopilot, as reflected by z and ϕ, can be used to determine the effectiveness of coefficients in both the longitudinal and lateral loops.

The degree of maximum bank angle change and steady-state bank angle late in a turn were used as a standard to compare the effect of the various coefficients as shown in figure 7.
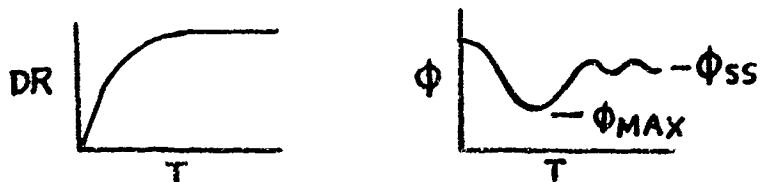
Figure 7. Bank Angle Comparison

Table 9 shows the relationship of ϕ max and ϕ_{ss} for several coefficients to the rudder deflection that was used.

TABLE 9. BANK ANGLE COMPARISON

Coefficient = 0	$\Delta\phi$ max%	$\Delta\phi_{ss}$ %
YARDR	10.0	11.0
YVAR	11.8	11.8
YVAV	13.7	14.7
KPAP	1.0	3.2
KVAV	-65.0	-70.0
NARDR	- 4.0	5.0
NAVR	3.5	- 3.5
NVAV	3.1	- 2.0

It is obvious that KVAV is necessary if the maximum and steady roll angles are to resemble the original effects at all. KVAV is multiplied by $v \cdot (v^2 + w^2)^{\frac{1}{2}}$ as in YVAV which has the second longest magnitude change. Therefore, XVAV can be utilized with only one additional multiplication and addition along with a data storage location for YVAV. NVAV was kept for the same reason although its effect is minor. This is because it can be added with very little effort and its sign is such that it aids in stabilizing YV when computing $\dot{\psi}$. This could be important during casualty conditions when the equations are making large changes.

Therefore, the lateral loop is reduced by setting YARDR, YVAR, KPAP, NARDR, and NAVR to zero. The effects of this did not cancel out, so

several runs were made to adjust KVAV to bring maximum bank angle and steady-state bank angles into line. In the case of the demonstration submarine, KVAV was reduced by 50%. It was found that heading angle was increasing at a faster rate than originally when all the modifications were made. Therefore, NDR was adjusted on a cut and try basis to bring heading rate back to its original value without affecting bank angle. NDR was reduced by 20% to achieve this effect.

Since KVAV is necessary and is to be adjusted no matter what happens to the other coefficients, it was concluded that this lateral set should be used for both the basic and modified set of equations.

6. CROSS-COUPLING COEFFICIENT REDUCTION

The cross-coupling coefficients are those that are used only when all six-degrees-of-freedom are energized. They are listed in table 10.

TABLE 10. NON-ZERO CROSS-COUPLING COEFFICIENTS

Axial	Lateral	Normal	Roll	Pitch	Yaw
XRR	YPQ	ZRR	KQR	MRR	NPQ
XRP	YWP	ZVR	KWP	MVR	NWP
XVR	YVW	ZVV		MVV	NVW
XVV		ZRP		MVP	
		ZVP		MRP	

All of these were set to zero and a number of steady-state turns were made under autopilot control. It was soon evident that something was wrong when the autopilot could not hold depth to within several hundred feet. When the original coefficients were used, the depth change was about four feet. A term-by-term check showed what the difficulty was:

TABLE 11. DEPTH CHANGE FOR STEADY TURN

Coefficient = 0	Change in $\overline{Z}$, ft.
None	4.16
XRR	5.29
XRP	5.21
XVR	9.81
XVV	4.63
ZRR	18.00
ZVR	12.38
ZVV	-33.00
ZRP	5.21
ZVP	4.53
MRR	-25.40
MVR	21.27
MVV	21.26
MVP	4.28
MRR	3.85

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ZRR, ZVR, ZVV, MRR, MVR, and MVV are required to make the equations behave like a submarine during steady-state turns. The other longitudinal cross-coupling coefficients and all of the lateral C-C coefficients can be set to zero. Any unbalance in the lateral loop can be taken care of with the KVAV adjustment.

7. THRUST TERMS

Thrust is computed as a function of the engine parameters in the general equations of simulation. This quantity is compared to the coefficient of drag, computed from aerodynamic parameters, in the axial force equation. When thrust equals drag, $\dot{u}$ is zero, and the forward body speed is in a steady-state condition. A slightly different method is used in the NSRDC equations of motion because the coefficient of drag does not appear explicitly in the axial equations.

Drag coefficients are computed as functions of rotational and side velocities through the water and as functions of the control surfaces, but all of these terms go to zero when the forward velocity lies along the X-body axis ($\alpha = 0$). The coefficient X_{uu} is not the main drag coefficient, but represents small departures of the drag coefficient from an equilibrium value. It is set to zero in the NSRDC equations.

Therefore, the main computation for the comparison between thrust and drag falls on the term

$$\frac{1}{2} \rho \ell^2 (a_i u^2 + b_i u u_c + c_i u_c^2)$$

where
ρ = water density
ℓ = submarine length
u = forward speed
u_c = command speed
a_i, b_i, c_i = thrust coefficients

a_i, b_i, and c_i are a set of constants that are used to represent propeller thrust and body drag. They are computed from both the steady-state thrust conditions and the acceleration-deceleration values for the submarine. Three sets are supplied with the submarine coefficients. One set is used when the commanded speed is half of the actual speed or greater. A second set is for below this value, and a third one is for full reverse. The three coefficients in each set must add up to zero. This is so that when commanded speed equals actual speed, thrust equals drag and the output of the above term is zero.

Acceleration and deceleration data was not available for the demonstration submarine. Therefore, it was assumed that the NSRDC coefficients accurately represented thrust and they were programed as indicated in the equations of motion. This means that the modified and basic equation sets are compared against the full set in acceleration-deceleration runs instead of using actual submarine data for verification.

In order to check out thrust response, program ZC300 was written to calculate both thrust in pounds and acceleration in feet per second squared for any condition of actual and commanded speed. The equation of interest is

$$m\dot{u} = \tfrac{1}{2}\rho\ell^2 (a_i u^2 + b_i u u_c + c_i u_c^2)$$

where m is the mass of the submarine. $m\dot{u}$ has the dimensions of pounds (English units) which is to be expected from $F = ma$. Therefore, the calculation of the term on the right-hand side of the equation can be printed out directly as pounds of thrust and, when divided by the mass of the submarine, as the acceleration in feet per second.

The equations of motion use the parameter η to indicate which set of coefficients to use in the thrust equation. This is merely the ratio of the commanded speed divided by the actual speed and indicates the break-points on the curves used to determine the coefficients. The thrust program allows η limits to be specified during input.

The commanded speed u_c is related to the RPM of the propeller by a linear constant. This constant can be determined from curves which are found in the NSRDC report "Resistance and Propulsion Characteristics of the _____ Submarine as Predicted from Model Tests" for the particular submarine of interest.

It should be noted that u_c can go negative and this can cause η to be negative. This happens when backing down during emergencies, and the coefficients are computed correctly for this. However, that does not mean that u can go negative. All of the coefficients assume that u is always positive and the thrust terms are calculated on this basis.

u_c can be calculated from RPM times a constant, but some additional subroutines would be required in a submarine simulator. The time history of an RPM change has to allow for communication lags, human response time, and engine response lags. This could be simulated by a delay followed by an exponential change to the new value of RPM.

8. KINEMATIC RELATIONS

The kinematic relations can be considered as a pair of transformations that will transform the body axis rates into Euler angle rates and the body axis velocities into earth axis velocities. Part of these transformations are given on page 12 in the NSRDC Report 2510, but not in a form that can be used conveniently for digital computation.

All forces and accelerations are calculated in the body axis system. This system lies along the physical axis of the submarine body and moves along with it. These are not the axes about which bank, pitch, and yaw rotate, and therefore a transformation is needed to calculate the angular rates. It is known as an Euler transformation and the development of

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this particular set is given in many text books[1]. NSRDC Report 2510 gives the following expression for bank, pitch, and yaw rates.

$$\dot{\phi} = p + \dot{\psi} \sin \theta$$

$$\dot{\theta} = \frac{q + \dot{\psi} \cos\theta \sin\phi}{\cos\phi}$$

$$\dot{\psi} = \frac{r + \dot{\theta} \sin\phi}{\cos\theta \cos\phi}$$

The trouble with this set of equations is that yaw rate has to be known to calculate pitching rate and pitching rate has to be known to calculate yaw rate. If the body axis angular rates are placed on one side of the equation, the Euler rates on the other, and matrix notation is used, the difficulty is obvious.

$$\begin{bmatrix} 1 & 0 & \sin\theta \\ 0 & \cos\phi & \cos\theta \sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

The above equation assumes that the Euler rates are known and the body axis rates desired when the reverse is true.

The inverse matrix is

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{\sin\theta \sin\phi}{\cos\theta} & \dfrac{\sin\theta \cos\phi}{\cos\theta} \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

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With this matrix, p, q, and r are calculated, and the transformation is made to solve for the Euler rates. An additional saving in complexity can be made if the new rates are calculated in the proper order. Expanding the matrix gives the following.

$$\dot{\phi} = p + \frac{\sin \Theta \, \sin \Phi}{\cos \Theta} \, q + \frac{\sin \Theta \, \cos \Phi}{\cos \Theta} \, r$$

$$= p + \sin \Theta \left\{ \frac{q \sin \Phi + r \cos \Phi}{\cos \Theta} \right\}$$

$$\dot{\psi} = \frac{q \sin \Phi}{\cos \Theta} + \frac{r \cos \Phi}{\cos \Theta} = \frac{q \sin \Phi + r \cos \Phi}{\cos \Theta}$$

Therefore, if $\dot{\psi}$ is calculated from q and r before the calculation for $\dot{\phi}$, the three equations can be written as follows.

$$\dot{\Theta} = q \cos \Phi - r \sin \Phi$$

$$\dot{\psi} = \frac{q \sin \Phi + r \cos \Phi}{\cos \Theta}$$

$$\dot{\phi} = p + \dot{\psi} \sin \Theta$$

The order of the calculations is important since a digital computer can evaluate only one term at a time. Notice that $\dot{\phi}$ is the same as the NSRDC set, but that the other two equations can be calculated independently. One problem with any Euler transformation is that singular points can occur. In the second equation, the determinate of the matrix is $\cos\Theta$, so the transformation is not valid at $\Theta = \pi/2$. The assumption is made that although large pitch angles $(\Theta > 60°)$ can occur, a submarine cannot get into a true vertical position. Therefore, the singularity should not give any trouble. As a practical matter, such a condition would give either the largest possible yaw rate, or the problem would stop due to a zero divide check error depending on the computer.

Generally, a maneuvering trainer is concerned with angles and depth, but not with position in the X-Y plane. Because of this, the NSRDC report gives an equation for vertical speed only related to the earth axis, and ignores the equations needed for X and Y positions. These equations were added to the research submarine program so that the outputs would be

complete. The only reason for including them in a submarine simulator program would be if it were to be used in a ballistic missile sub-trainer where position with reference to distant points is important. The transformation used changes the body axis velocities to earth axis velocities with x along the north axis, y along the east axis, and z downward. The equations are

$$\dot{Z} = -u \sin\Theta + v \cos\Theta \sin\Phi + w \cos\Theta \cos\Phi$$

$$\dot{X} = u \cos\Theta \cos\Psi + v(\sin\Phi \sin\Theta \cos\Psi - \cos\Phi \sin\Psi)$$
$$+ w(\cos\Phi \sin\Theta \cos\Psi + \sin\Phi \sin\Psi)$$

$$\dot{Y} = u \cos\Theta \sin\Psi + v(\sin\Phi \sin\Theta \sin\Psi + \cos\Phi \cos\Psi)$$
$$+ w(\cos\Phi \sin\Theta \sin\Psi - \sin\Phi \cos\Psi)$$

It can be seen that time and space will be saved in the computer if x and y do not have to be calculated.

There exist fairly simple algorithms for calculating sine and cosine functions. The simplest one of all is suggested for the basic trainer if core space problems require that the final program be as small as possible. They are

$$\sin\Theta = 0.971 \ \Theta \text{ radians}$$

$$\cos\Theta = 1.02 - 0.28 \ \Theta \text{ radians}$$

These functions have less than 5% error up to 42°. Submarine pitch angles can exceed this limit, but it should not occur too often in a basic maneuvering trainer.

A more satisfactory algorithm for the modified simulation is

$$\sin\Theta = \Theta - \frac{\Theta^3}{6}$$

$$\cos\Theta = 1 - \frac{\Theta^2}{6} + \frac{\Theta^4}{24}$$

They have less than 3% error at 75° which should serve for any purpose. Of course, both work only over the range of $\pi/2$ to $-\pi/2$ so sin ψ and cos ψ will have to be calculated by a more complex subroutine if X and Y are to be calculated.

9. NEAR SURFACE OPERATION

Two effects come into play when a submarine is operating close to the surface. The coefficients of the equations of motion are changed due to the presence of the surface and forces and moments are generated by waves on the surface.

An investigation was made into the effects of coefficient changes on the demonstration submarine. These coefficients were supplied by NSRDC as a result of depthkeeping studies under a seaway and are assumed to be correct. The reasons for these changes are that a body moving under the free surface of a liquid produces a flow pattern which in turn affects the body itself. In addition, the nature of the surface in the form of wind waves influences the body. The magnitude of all of these effects is related to the depth of the body below the surface and its velocity.

When the body is near the surface, the flow pattern produced in between the body and the surface is a Bernoulli contour system. The positive pressures at the bow and stern produce wave crests, while the lower pressure at mid-body produces a trough. The reduced depth over the mid-portion of the body reduces the streamline flow area between body and surface. The establishment of the Bernoulli contour system increases the positive pressures at bow and stern and decreases the mid-body pressure on the upper surface of the body. Since this effect is not produced on the bottom at relatively low velocities, the low pressures act over a larger area on the top surface resulting in a net vertical force on the body acting to push the body toward the surface.

As the velocity of the body increases, the wave pattern on the surface lags more and more, resulting in changes to the pressure pattern to the extent that the positive pressure in the bow area covers more and more of the upper area. A net transverse force in the downward direction is then produced. Because of the Bernoulli contour system effect, the positive pressure at the bow is greater while the positive pressure at the stern is lower. This condition results in an increase in pressure drag, varying with body velocity and depth, and known generally as wave resistance.

One of the references indicated that the change in damping ratio, in particular, could be substantial under near-surface conditions[9], but another one indicated the opposite[10]. Therefore, impulse runs were made using coefficients modified for the above effect and were compared with the deep coefficients.

Coefficients were available at 6 and 12 knots, so impulse runs were run at these speeds. Pitch angle was recorded as a typical parameter. The two runs for deep and surface coefficients were plotted

against each other. A shift in the vertical axis was due to different initial conditions of level flight, and the result shows very little difference in the response at either 6 or 12 knots. In each case, the "initial conditions" program EC470 was used to generate a neutral pitch angle, normal velocity, and sternplane deflection for level flight.

Because the critical speed for the demonstration submarine is below six knots, it was decided to repeat an impulse run at two knots to ensure that performance did not change at very low speeds. Coefficients were not available at this speed, so the values for 6 and 12 knots were extrapolated into the 2 knot area. The main changes were in ZW and MW which were lowered by 3% and 15% respectively. This run had a minor phase shift that would not be noticeable to the trainee, but was otherwise identical to the deep coefficient case at two knots.

The results of the impulse runs indicate that the handling qualities of the demonstration submarine do not change appreciatively due to the presence of a calm near-surface. A further check was made using overshoot runs as a criteria. The change in forward speed, pitch angle, and depth at values of maximum change between the two sets of coefficients are given in table 12.

TABLE 12.   PERCENT CHANGE OF CHANGE, DEEP TO SURFACE COEFFICIENTS

	6 Knots	12 Knots
Δu	7.7	13.0
Δe	-1.1	3.7
ΔZ	-4.3	10.9

Because the change in u is very small during these overshoot runs, the 13% difference represents an absolute change of less than one tenth of a knot at twelve knots. A comparison of the curves of the runs shows that very little change occurs when surface coefficients are used.

The one thing that might bother the trainee is that the conditions for level flight are different close to the surface. For example, the neutral pitch angle becomes slightly negative as the surface is approached. It was also felt that much of the change in overshoot response was due to this difference, so a run was made at twelve knots and used surface coefficients except for Z_* and M_*. The changes in this case are seen in table 13.

In other words, most of the change during an overshoot run was due to the different M_* and Z_* determining different neutral angles and not due to the use of other surface coefficients.

Therefore, it will not be necessary to use surface coefficients in either trainer. If very accurate near-surface operation is required as part of the training mission, M_* and Z_* should be computed as a function

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TABLE .. PERCENT CHANGE, SURFACE COEFFICIENTS

Parameters at 12 Knots	% Change
Δu	3.6
$\Delta \Theta$	1.1
ΔZ	1.3

of velocity and depth in the modified program.

The values of the wave coefficient magnitudes were not supplied by NSRDC although work has been done in this area. An investigation was made into the magnitude of these coefficients, during a Sea State 5 sea using the classical methods of hydrodynamics[--].

Calculations for the wave surface amplitude have been calculated in the section on Wave Generation. The forces and moments are resolved through the difference between the heading angle of the submarine and the heading angle of the waves. Usually, a simplification is made that the wind is always blowing from the north, but a variable wind direction can be added easily. The forces and moments are then varied as a function of the depth, reaching a negligible value at about five hull diameters below the surface.

D. MODIFIED AND BASIC PROGRAM SIMULATION

The modified submarine math model can be written when all the approximations to the full-set equations of motion have been determined. The first step is to set all negligible coefficients to zero and remove the parameters not being used. The equations are solved for the six accelerations, and any set of terms that can be reduced to a constant is replaced by that constant. A certain amount of juggling is performed in order to reduce the number of arithmetical operations, and the final result is known as the compact submarine simulation program. This is the program that would be used in a submarine simulator irrespective of whether analog or digital computation was used and, in the case of a digital computer, whether or not FORTRAN or assembly language was used. The program for six-degrees-of-freedom movement, Compact Equations of Motion (EC780), are given in equations (12) through (17).

Axial   (12)

$$\dot{u} = (+ M\,(X_G\,(q^2 + r^2) - Z_G\,(pr + \dot{q}) + vr - wq)$$

$$+ u^2\,(X_{\delta r \delta r}\,\delta_r^2 + X_{\delta s \delta s}\,\delta_s^2 + X_{\delta b \delta b}\,\delta_b^2 + a_i)$$

$$+ u_c\,(b_i\,u + c_i\,u_c) - (W - B)\sin\Theta)/(M - X_{\dot{u}})$$

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Lateral

(13)

$$\dot{\wr} = \Big(u(Y_{ur}r + Y_p p + Y_{\wr}\wr)$$

$$-M_{..}(Z_G(qr - \dot{p}) + X_G(qp + \dot{r}) - wp + ur\Big)$$

$$+ Y_{\dot{r}}\dot{r} + Y_{\dot{p}}\dot{p} + Y_{\wr|\wr|}\wr(\wr^2 + w^2)^{\frac{1}{2}}$$

$$+ Y_{\delta_r}u^2\delta_r + (W-B)\cos\theta\sin\phi\Big)\Big/\Big(m - Y_{\dot{\wr}}\Big)$$

Normal

(14)

$$\dot{w} = u(Z_q q + Z_w w)$$

$$+ M (Z_G(p^2 + q^2) - X_G(rp - \dot{q}) + uq - \dot{y}p)$$

$$+ Z_{\dot{q}}\dot{q} + Z_{rr}r^2 + Z_{\wr r}\wr r + Z_{\wr\wr}\wr^2$$

$$+ Z_{w|w|}w(\wr^2 + w^2)^{\frac{1}{2}}$$

$$+ u^2(Z_{\delta_s}\delta_s + Z_{\delta_b}\delta_b + Z_*)$$

$$+ (W-B)\cos\theta\cos\phi\Big)\Big/\Big(m - Z_{\dot{w}}\Big)$$

Roll

(15)

$$\dot{p} = m_. Z_G(\dot{\wr} - wp + ur) + K_{\dot{p}} K_{\dot{r}}\dot{r} + K_{\dot{\wr}}\dot{\wr}$$

$$+ u(K_p p + K_{\wr}\wr) + K_{\wr|\wr|}\wr(\wr^2 + w^2)^{\frac{1}{2}}$$

$$- K_{\dot{p}}(Z_G W - Z_\bullet B)\cos\theta\cdot\cos\phi$$

Pitch

(16)

$$\dot{q} = M_{rp}rp + M_{\dot{w}}\dot{w} + u(M_q q + M_w w)$$

$$- m (Z_G(\dot{u} - \wr r + wq) - X_G(\dot{w} - uq + \wr p))\cdot M_{\dot{q}}$$

$$+ M_{rr}r^2 + M_{\wr r}\wr r + M_{\wr\wr}\wr^2$$

$$+ M_{|\wr|q}q(\wr^2 + w^2)^{\frac{1}{2}} + M_{w|w|}w(\wr^2 + w^2)^{\frac{1}{2}}$$

$$+ u^2(M_* + M_{\delta_b}\delta_b + M_{\delta_s}\delta_s)$$

$$- M_{\dot{q}}(X_G W\cos\theta\cos\phi + (Z_G W - Z_\bullet B)\sin\theta)$$

Yaw   (17)

$$\dot{r} = N_{pq} pq + N_{\dot{p}} \dot{p} + N_{\dot{w}} \dot{w} + u^2 N_{\delta_r} \delta_r$$
$$- m \cdot X_G (\dot{w} - wp + ur) N_{\dot{r}} + N_{w|w|} N (w^2 + w^2)^{\frac{1}{2}}$$
$$+ u (N_p p + N_r r + N_w w) + N_{\dot{r}} (X_G W) \cos\theta \sin\phi$$

The parameters are defined in equation (18). In the equations that follow, the terms on the left side of the equation are used in the compact submarine simulation and the terms on the right side are the original NSRDC coefficients as defined in NSRDC Report 2510. The only exception to this is T_{l_1}, T_{5}, etc. which are used on both sides of the equation once the terms for the inertial mass has been inverted. These values are different in the two programs, because one is normalized and the other is not, but they employ the same symbology because they occupy the same locations in the two equations of motion.

Coefficient Description (EC780)   (18)

$$XORDR' = X'_{\delta_r \delta_r} = \frac{\rho}{2} \ell^2 X_{\delta_r \delta_r}$$

$$XDSDS' = X'_{\delta_s \delta_s} = \frac{\rho}{2} \ell^2 X_{\delta_s \delta_s}$$

$$XDBDB' = X'_{\delta_b \delta_b} \approx \frac{\rho}{2} \ell^2 X_{\delta_b \delta_b}$$

$$A11' = a'_{i1} = \frac{\rho}{2} \ell^2 a_{i1}$$

$$A12' = b'_{i1} = \frac{\rho}{2} \ell^2 b_{i1}$$

$$A13' = c'_{i1} = \frac{\rho}{2} \ell^2 c_{i1}$$

$$A21' = a'_{i2} = \frac{\rho}{2} \ell^2 a_{i2}$$

$$A22' = b'_{i2} = \frac{\rho}{2} \ell^2 b_{i2}$$

$$A23' = c'_{i2} = \frac{\rho}{2} \ell^2 c_{i2}$$

$$A31' = a'_{i3} = \frac{\rho}{2}\ell^2 a_{i3}$$

$$A32' = b'_{i3} = \frac{\rho}{2}\ell^2 b_{i3}$$

$$A33' = c'_{i3} = \frac{\rho}{2}\ell^2 c_{i3}$$

$$XUD' = X'_{\dot{u}} = \frac{\rho}{2}\ell^3 X_{\dot{u}}$$

$$YR' = Y'_r = \left(\frac{\rho}{2}\ell^3 Y_r - m\right)$$

$$YRD' = Y'_{\dot{r}} = \frac{\rho}{2}\ell^4 Y_{\dot{r}}$$

$$YPD' = Y'_{\dot{p}} = \frac{\rho}{2}\ell^4 Y_{\dot{p}}$$

$$YP' = Y'_p = \frac{\rho}{2}\ell^3 Y_p$$

$$YV' = Y'_v = \frac{\rho}{2}\ell^2 Y_v$$

$$YVAV' = Y'_{v|v|} = \frac{\rho}{2}\ell^2 Y_{v|v|}$$

$$YDR' = Y'_{\delta r} = \frac{\rho}{2}\ell^2 Y_{\delta r}$$

$$YVD = Y'_{\dot{v}} = \frac{\rho}{2}\ell^3 Y_{\dot{v}}$$

$$ZQ' = Z'_q = \frac{\rho}{2}\ell^3 Z_q$$

$$ZQD' = Z'_{\dot{q}} = \frac{\rho}{2}\ell^4 Z_{\dot{q}}$$

$$ZRR' = Z'_{rr} = \frac{\rho}{2}\ell^4 Z_{rr}$$

$$ZVR' = Z'_{Nr} = \frac{\rho}{2}\ell^3 Z_{Nr}$$

$$ZSTR' = Z'_* = \frac{\rho}{2}\ell^2 Z_*$$

$$ZW' = Z'_\omega = \frac{\rho}{2}\ell^2 Z_\omega$$

$$ZWAW' = Z'_{\omega|\omega|} = \frac{\rho}{2}\ell^2 Z_{\omega|\omega|}$$

$$ZVV' = Z'_{NN} = \frac{\rho}{2}\ell^2 Z_{NN}$$

$$ZDS' = Z'_{\delta s} = \frac{\rho}{2}\ell^2 Z_{\delta s}$$

$$ZDB' = Z'_{\delta b} = \frac{\rho}{2}\ell^2 Z_{\delta b}$$

$$ZWD' = Z'_{\dot\omega} = \frac{\rho}{2}\ell^3 Z_{\dot\omega}$$

$$T_4 = 1/(I_x - \frac{\rho}{2}\ell^5 K_{\dot p})$$

$$AKRD' = K'_{\dot r} = \frac{\rho}{2}\ell^5 K_{\dot r} \cdot T_4$$

$$AKP' = K'_p = \frac{\rho}{2}\ell^4 K_p \cdot T_4$$

$$AKVD' = K'_{\dot N} = \frac{\rho}{2}\ell^4 K_{\dot N} \cdot T_4$$

$$AKV' = K'_{N} = \frac{\rho}{2}\ell^3 K_{N} \cdot T_4$$

$$AKVAV' = K'_{N|N|} = \frac{\rho}{2}\ell^3 K_{N|N|} \cdot T_4$$

$$AKPD' = K'_{\dot p} = T_4$$

$$T_5 = 1/(I_Y^{'} - \frac{\rho}{2}\ell^5 M_{\dot{q}})$$

$$AMRP' = M_{rp}^{'} = (I_z - I_x + \frac{\rho}{2}\ell^5 M_{rp})\cdot T_5$$

$$AMRR' = M_{rr}^{'} = \frac{\rho}{2}\ell^5 M_{rr}\cdot T_5$$

$$AMWD' = M_{\dot{w}}^{'} = \frac{\rho}{2}\ell^4 M_{\dot{w}}\cdot T_5$$

$$AMVR' = M_{vr}^{'} = \frac{\rho}{2}\ell^4 M_{vr}\cdot T_5$$

$$AMQ' = M_{q}^{'} = \frac{\rho}{2}\ell^4 M_{q}\cdot T_5$$

$$AMAWQ' = M_{|w|q}^{'} = \frac{\rho}{2}\ell^4 M_{|w|q}\cdot T_5$$

$$AMSTR' = M_{*}^{'} = \frac{\rho}{2}\ell^3 M_{*}\cdot T_5$$

$$AMW' = M_{w}^{'} = \frac{\rho}{2}\ell^3 M_{w}\cdot T_5$$

$$AMWAW' = M_{w|w|}^{'} = \frac{\rho}{2}\ell^3 M_{w|w|}\cdot T_5$$

$$AMVV' = M_{vv}^{'} = \frac{\rho}{2}\ell^3 M_{vv}\cdot T_5$$

$$AMDS' = M_{\delta s}^{'} = \frac{\rho}{2}\ell^3 M_{\delta s}\cdot T_5$$

$$AMDB' = M_{\delta b}^{'} = \frac{\rho}{2}\ell^3 M_{\delta b}\cdot T_5$$

$$AMQD' = M_{\dot{q}}^{'} = T_5$$

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$$T_6 \qquad\qquad = \quad 1/(I_z - \tfrac{\rho}{2}\,\ell^5 N_{\dot{r}})$$

$$ANPQ' \;=\; N'_{pq} \quad = \quad (I_x - I_y + \tfrac{\rho}{2}\,\ell^5 N_{pq})\cdot T_6$$

$$ANPD' \;=\; N'_{\dot{p}} \quad = \quad \tfrac{\rho}{2}\,\ell^5 N_{\dot{p}}\cdot T_6$$

$$ANVD' \;=\; N'_{\dot{v}} \quad = \quad \tfrac{\rho}{2}\,\ell^4 N_{\dot{v}}\cdot T_6$$

$$ANP' \;=\; N'_{p} \quad = \quad \tfrac{\rho}{2}\,\ell^4 N_{p}\cdot T_6$$

$$ANR' \;=\; N'_{r} \quad = \quad \tfrac{\rho}{2}\,\ell^4 N_{r}\cdot T_6$$

$$ANV' \;=\; N'_{v} \quad = \quad \tfrac{\rho}{2}\,\ell^3 N_{v}\cdot T_6$$

$$ANVAV' \;=\; N'_{v|v|} \quad = \quad \tfrac{\rho}{2}\,\ell^3 N_{v|v|}\cdot T_6$$

$$ANDR' \;=\; N'_{\delta r} \quad = \quad \tfrac{\rho}{2}\,\ell^3 N_{\delta r}\cdot T_6$$

$$ANRD' \;=\; N'_{\dot{r}} \quad = \quad T_6$$

$$ZB' \;=\; \mathfrak{z}'_0 \quad = \quad CB\cdot\mathfrak{z}_0$$

Since MRP and NPQ can be added to the compact simulation at no additional cost, these terms were kept even though their necessity is doubtful.

Although the compact coefficients could be calculated by hand, it is far easier to use the computer. This is especially true considering that the input deck for program EB920 contains all of the necessary NSRDC coefficients. The resulting values will have to be punched on cards as input to the compact submarine simulation program.

Program EC790 - Compact Coefficient Calculator was written to perform this chore. The program consists of a main program and two subroutines called INPUT and WRITE. The main program calls INPUT to read in a deck of cards and then calls WRITE to write them out. The seventy or so new coefficients are calculated and WRITE is called again to write out the new values. These new values are then punched on cards in a format t? can be used as an input to the compact submarine simulation program.

Subroutine INPUT was taken directly from the main program of EB920. It reads in the same data deck as EB920 including the values that are not going to be used. This saves having to make up a new input deck to use this program after EB920 has been used in the investigation of a set of coefficients.

Subroutine WRITE was also taken from EB920. It uses the same output format for printing out the coefficients. When the program prints out the same format for the second time, the values have become those on the output cards. These are then used in the compact simulation program even though the nomenclature remains the same. Values not used by the new program remain the same.

Program EC780 - Compact Submarine Simulation is a FORTRAN program using the mathematical model for the reduced equations of motion. It represents the chief goal of the study in that it is as small as possible while still simulating the path of a submarine accurately.

This program can be run in any digital computer that has a FORTRAN compiler and enough core. It has been run on the IBM 360/40, SDS Sigma 5, and an SDS Sigma 2 which is a small 16-bit machine. In every case, it gave good results when compared against EB920 with the same set of input conditions.

The MAIN program reads in the input cards produced by EC790, prints out a heading called UPDATE, prints out the twelve parameters of velocity and position and then checks the time to see if the program is done.

The UPDATE subroutine is equivalent to a combined EVAL and INTEG subroutine in EB920. It calculates velocities, thrust, trig functions, accelerations, Euler angles, and then integrates using 2nd order Adams. This program is all that is necessary for the flight portion of a submarine simulator.

Subroutine CONTR is identical to the same subroutine in EB920.
It was added so that the compact program would be checked out against
EB920 and no attempt was made to shorten it. It allows climbing turns,
flat turns, meanders, overshoots, and impulse runs in the same manner as
the original.

Finally, the output format for EC780 was designed so that the twelve
variables could be printed out on a teletypewriter rather than a line
printer. This is so that a very small computer can be used and useful
results can still be obtained.

E. WAVE GENERATION

The analytical treatment of wind-generated ocean waves has undergone
a revolution in the last five years. This started in 1953 when Dr. Pierson
and Dr. Neumann published "Practical Methods for Observing and Forecasting
Ocean Waves by Means of Wave Spectra and Statistics". The field was
entirely shifted over from a determinate one to a statistical basis by
1963 with the publication of "Ocean Wave Spectra"[12]. This was an account
of proceedings of the Easton, Md. conference in that year. The field is
a complex one and, rather than giving a superficial treatment here, this
section will attempt to outline some of the references that lead to an
understanding of wave spectra. This will be followed by a discussion of
two different methods of generating wave amplitude with a digital computer
program.

The basic reference is Chapter 8 of "Wind Waves"[13] by Blair Kinsman.
This chapter gives a short history of the development of wave spectra
theory, discusses some of the stationary Gaussian process mathematical
models, shows the development of the Neumann spectrum, and finally indicates
the uses to which the Pierson - Neumann theory can be put. Some of the
mathematics is fairly rough going, but the text is very clear and under-
standable.

The second reference that is a "must" is "Sea Spectra Simplified"[14]
by Walker H. Michel. This report discusses the concept of a wave spectrum
from a physical point of view and shows why a frequency spectrum is used,
written with a minimum of mathematics. More importantly, it gives a
review of the different spectra that have been developed in the past few
years and outlines the notation and units used by the different workers
in the field. Just to illustrate one source of confusion, some use energy,
some use wave height, and some use amplitudes. Finally, ten references
are given that cover the development of the wave spectra theory.

The first nine pages of "Recent Developments in Seakeeping Research
and its Application to Design"[15] give a brief discussion of the same
areas as Michel's paper. The end of the paper has a list of 142 references
covering almost every paper of the "statistical era" since 1960. It is
available as a reprint from SNAME.

The three references listed give a good overall view, but they do
not tie down an actual spectrum that could be used during a wave generation

investigation. This is done in "A Sea Spectrum for Model Tests and Long-Term Ship Prediction"[16] by J.R. Scott. A particular spectrum is given and justified along with the transformation that determines the energy in the spectrum proportional to either the average wave height (sea state) or the wind speed existing at the place where the waves are assumed to exist.

All of these references indicate how to measure waves, determine the spectrum characteristics, and apply the theoretical results obtained; but they do not give any help as to how a mathematical model might be developed to generate wave amplitudes with a computer. The two different approaches are covered in "The Analysis and Modeling of Irregular Waves"[17] and "Mathematical Generation of a Realistic Sea"[18] by Chen et al. The second report is out of print, but it may be obtained from the Defense Documentation Center under number AD609906 by qualified users. A review of these papers and references will give a complete background into the subject of statistical wave generation for research purposes.

However, no one in the field seems to be using these techniques with a digital computer to produce waves with the desired statistical properties in real-time. The closest approach is in the wave-making machine used with a towing tank for testing uses. In this case, signals are generated using an analog computer and recorded in analog form on magnetic tape. The tape is played back when a wave signal is desired. The two different methods were examined, a mathematical model was developed, and a computer program was written to generate a random wave over time.

The concept of a wave spectrum is based on the energy content of the wave, which is proportional to the sea state, being divided into different frequencies of superimposed sinusodial waves. The energy divides according to hydrodynamic principles into a particular frequency curve which can be determined by making many measurements of waves and analyzing the resulting data. The irregular surface is due to the fact that these sinusoids have phase angles which occur in a completely random manner with time. Therefore, they add and subtract to present the typical irregular surface seen on the ocean, rather than the regular wave-form expected from a trigonometric Fourier expansion with fixed phases.

The energy in a sine wave is related to the amplitude by the expression

$$E = \rho g \frac{h^2}{8}$$

where ρ is the water density and h is the wave height.

To calculate a wave height at a particular point, a sine wave is calculated for each of a number of frequencies. Their amplitude, and therefore their energy, at each frequency is determined by the wave spectra curve. These frequencies are shifted in random phase and added together for each time point. The main difference between the two methods outlined

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is how the frequency division is made. The older of the two methods is the addition of a finite number of cosine waves of different amplitudes and frequencies. The following mathematical model was developed from "Mathematical Generation of a Realistic Sea" and section 8.3 of "Wind Waves". The surface elevation can be represented as a stationary Gaussian process in three dimensions. This can be reduced to a function of time by assuming that only the single point directly over the submarine is of interest, and placing the coordinate reference system origin at this point. The surface elevation then becomes

$$\zeta(t) = \int_0^\infty \int_{-\pi}^{\pi} \cos\left[\omega t + \epsilon(\omega, \beta)\right] \sqrt{A^2(\omega, \beta) \, d\omega \, d\beta}$$

where

ω = frequency

β = difference between wind direction and wave direction

$\epsilon(\omega, \beta)$ = random phase angle ($0-2\pi$)

$A^2(\omega, \beta)$ = wave spectrum

The reason for putting the differential under the radical sign is covered in Chapter 8 of "Wind Waves". In order to program this integral on a digital computer, it was necessary to divide it into discrete elements so that they can be added numerically. This is a double integral so the division has to be made in both ω and β.

The first task is to divide the directional dependence energy spectrum into equal bands on each side of the primary wind direction. This is usually assumed to be

$$A^2(\omega, \beta) = \begin{cases} \dfrac{2}{\pi} A^2(\omega) \cos^2\beta \; ; & -\dfrac{\pi}{2} \le \beta \le \dfrac{\pi}{2} \\ 0 \; ; & \text{otherwise} \end{cases}$$

This converts the spectrum into a function of frequency only and allows separate calculations of the β part of the integral. We then can divide the integral according to the example.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\beta \, d\beta$$

This now has five equal energy bands off the main wind axis by using constants evaluated in table 14 as calculated in Chen's report.

TABLE 14. WAVE ENERGY BANDS

β	$\Delta\beta$ SECTOR	$a_i = \int \cos^2\beta \, d\beta \mid$ SECTOR
$-60°$	$-90°$ to $-52°$	0.0906
$-30°$	$-52°$ to $-15.4°$	0.4330
$0°$	$-15.4°$ to $+15.4°$	0.5325
$30°$	$15.4°$ to $52°$	0.4330
$60°$	$52°$ to $90°$	0.0906

a_i represents the energy in a sector which contains β dividing the sector into two equal areas on each side of β .

The wave spectrum is also divided into equal energy areas. The total area under the spectrum curve represents total energy in the wave and it is a simple job to solve for the frequency points which give equal areas, after integrating the function for $A^2(\omega)$. If each of these points is ω'_j ; then

$$\omega_j = \frac{\omega'_j - \omega'(j-1)}{2}$$

$$\Delta\omega_j = \omega'_j - \omega'(j-1)$$

We can now write the surface elevation as a summation of discrete points

$$S(t) = \sqrt{\frac{2}{\pi}} \sum_{i=1}^{S} \sum_{j=1}^{N} \cos(\omega_j t + \epsilon_{ij}) \sqrt{A^2(\omega_j) a_i \, \Delta\omega_j}$$

where all the terms have been defined.

This mathematical model was programed using the Pierson - Moskowitz spectrum. This is a spectrum curve that is being used as the interim standard in both analytical and experimental studies. Its equation is

$$A^2(\omega) = \frac{8.1 \times 10^{-3} g^2}{\omega^5} e^{-.74 \left(\frac{g}{U\omega}\right)^4}$$

where U is the wind speed. The actual equation programed was

$$S(t) = \sqrt{\frac{16.2 \times 10^{-3} \times \overline{32.2}^2}{\pi}} \sum_{j=1}^{10} \sum_{i=1}^{5} \cos(\omega_j t + \epsilon_{ij})$$

$$\left[\left(\omega_j^{-5} e^{-.74 \left(\frac{32.2}{U}\right)^4 \omega_j^4} \right) (a_i \Delta \omega_j) \right]^{\frac{1}{2}}$$

ω_j had ten different values which were computed from the spectrum and ran from a lower limit of 0.2 rad/sec. to 1.8 rad/sec. U was chosen to be 40 feet per second and the a_i's were read in from cards. A random number generator was called fifty times and scaled for 0 to 2π to calculate values for ϵ_{ij}. These values were computed and stored each time the program was used. Figure 8 shows 400 points calculated at one second intervals. The Y axis is in per cent of the largest point calculated. This would correspond to a wave height of 25 ft. at this wind speed.

This program takes about one-tenth of a second to calculate one point in time and requires quite a bit of core space to hold the values used to make up the summation (fifty in the case of the example), so it is useful only for the research program. It would have to be greatly simplified for use in a submarine simulator.

The second method was recently borrowed from communications and filter theory. It consists of a filter network that has a frequency response that is equivalent to the desired spectral density energized by a "white noise" random excitation. "White noise" has a constant spectral density containing all frequencies equally, and since a filter in the frequency domain can be considered as a variable gain device, the output will have frequencies statistically equal to that of ocean waves. White noise cannot be realized as a practical matter, but ocean wave spectra are limited to 2 radians per second, so a flat band-limited signal will do. This is a very easy process when using an analog computer. A gas tube is used as a white noise source and a filter is designed and programed using conventional techniques. The job is a little harder when using a

digital computer.
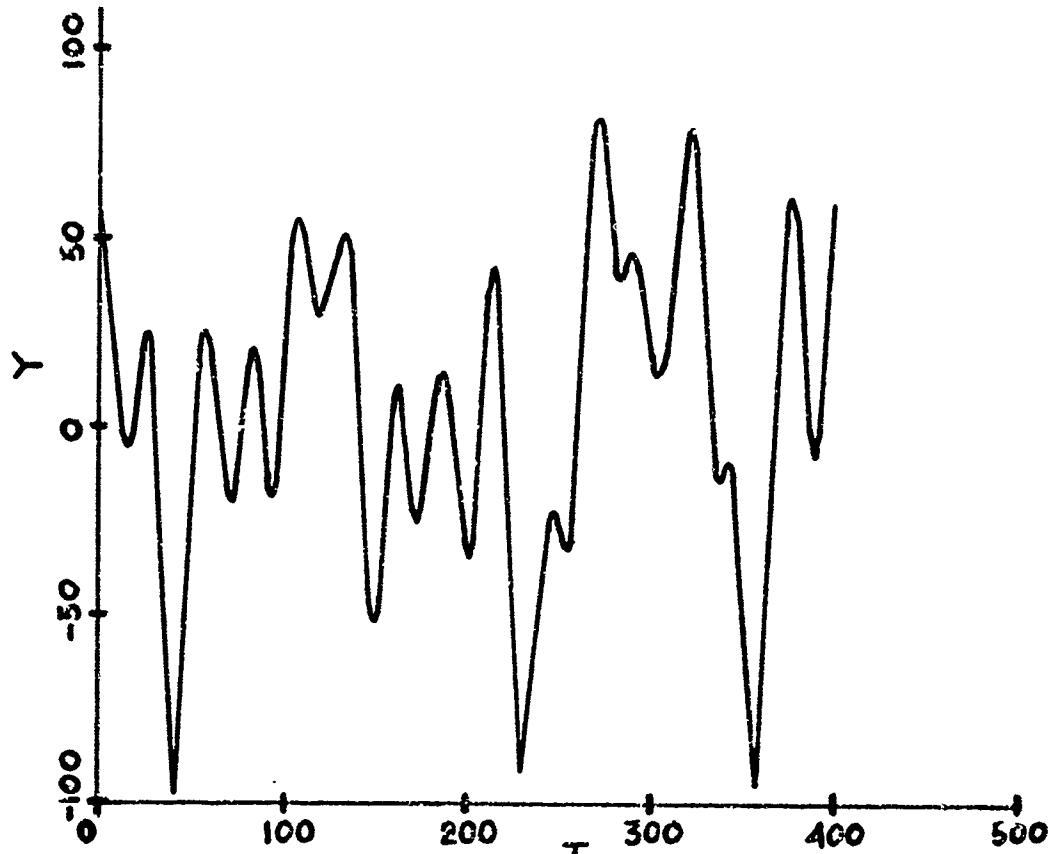
Figure 8. Typical Wave Output

The "Analysis and Modeling of Irregular Waves" gives a mathematical model that can be used as a filter to create the Pierson - Moskowitz spectrum. The desired frequency response for the filter is

$$H(\omega) = \left(A^2(\omega)\right)^{\frac{1}{2}} = \frac{.098}{\omega^{5/2}} \; e^{-.37\left(\frac{g}{U\omega}\right)^4}$$

and the frequency response can be duplicated by means of a low pass filter followed by a three section high pass filter. The constants were derived empirically by plotting $H(w)$ and comparing the curve with the filter frequency response as the damping factor and natural frequency of each section was varied. The filter equation is

$$H(s) = \frac{1}{s^2 + 2\delta_L W_{n_L} S + W_{n_L}^2} \left(\frac{s^2}{s^2 + 2\delta_H W_{n_H} S + W_{n_H}^2}\right)^3$$

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TABLE 15. WIND CONSTANTS

Wind Speed (knots)	W_{h_L}	δ_L	W_{h_H}	δ_H
45	0.39	0.325	0.21	0.707
40	0.43	0.25	0.24	0.707
35	0.48	0.325	0.27	0.707

The factors were given in the reference and were used in a digital computer program to generate waves for the 40 knot case. This program was not included in the study library of programs because it uses a noise generator that is both IBM 360/40 machine-dependent and GAC DOS Operating System dependent, and therefore will run only at the Goodyear Aerospace facility. A white noise generator that operates as fast as possible and has a Gaussian probability distribution function would be written for the particular computer used.

The programing of the filter is straight forward. It is necessary to find a set of difference equations having a system function $R(z)$ which is equivalent to that of $H(s)$. A method due to Baxter[6] is given.

If a transfer function is written in terms of s, the Laplace operator, the function $1/s$ can be considered as an integration. Therefore, if an integration z-transform $I(z)$ is known, the filter z function can be written by replacing the s with the factor $1/I(z)$. An example will make this clear. Assume

Filter

$$H(s) = \frac{1}{s^2 + 2 W_h \delta S + W_h^2}$$

Euler Integration 0_{11} with time integral h

$$I(z) = \frac{h Z^{-1}}{1 - Z^{-1}}$$

Thus $H(z)$ equals $H(1/I(z))$ which equals

$$\frac{1}{\left(\frac{1-Z^{-1}}{h Z^{-1}}\right)^2 + 2 W_h \delta \frac{(1-Z^{-1})}{h Z^{-1}} + W_h^2}$$

or

$$\frac{h^2 z^{-2}}{1-(2+2W_h\,\delta\,h)Z^{-1}+(1-2W_h\,\delta h+W_h^2 h^2)Z^{-2}}$$

This can be written directly as a difference equation through the method outlined in the section on integration methods and can then be programed into

$$Y_n=(2+2W_h\,\delta\,h)Y_{n-1}-(1-2W_h\,\delta h+W_h^2 h^2)Y_{n-2}+h^2\ddot{Y}_{n-2}$$

The same method is used with the other sections of the filter. The first $\ddot{Y}_{n-2}$ is the output of the random noise generator and each Y_n becomes the $\ddot{Y}$ of the next section.

This method worked as well as the cosine series addition because there were only 16 multiplication and additions along with the generation of the random number. It should be noted that most Gaussian random-number generators are extremely slow and are not suitable for real-time. The one used at GAC is written in IBM machine language in order to perform shifting operations that are difficult to do in FORTRAN. This is not a problem with the cosine generator because the random numbers are generated only when the subroutine is first entered.

Since this work was done, another reference on direct programing of Laplace transfer functions was found. The reference "Digital Filter Design Techniques in the Frequency Domain" by Rader and Gold gives several methods. One that has been utilized for filter design on another project at Goodyear Aerospace is as follows. Replace the natural frequency of the filter by a new natural frequency

$$W_L(New) = \tan\frac{W_L h}{2}$$

where h is the time interval of the difference equation to be used. Then replace s by $(z-1)/(z+1)$ in H(s) and perform the algebra necessary to express the resulting H(z) as a ratio of polynomials. The difference equation[19] can then be written directly.

F. SMALL COMPUTER UTILIZATION

The utilization of a small general purpose computer was investigated once the characteristics of the compact submarine simulation program were known. This was accomplished in the most direct manner possible; namely, by running the compact program in a 16-bit machine and examining the results.

There is no doubt that a 16-bit computer can be used in a submarine simulator. Accuracy is not a problem if a higher level language such as FORTRAN is used because of the way a 16-bit machine utilizes floating point numbers. Two words are used to store data. The first word contains the exponent and the first six bits of the fraction. The second word contains the rest of the fraction so that a 24-bit word plus exponent is actually being used. This allows seven figure decimal accuracy to be obtained in the simulation computations.

The computer used for running the compact simulation program was an SDS Sigma 2. It has a 2.25 μ-sec add and cycle time which is typical of this class of computers. It did not have either floating point hardware or fixed point multiply-divide hardware, so any programs represent a worst case condition of slowest speed and maximum core size.

The compact program was run on the IBM 360/40 (32-bit word) to produce a check program against the Sigma 2. The output format in each case was six decimal figures plus exponent, and the two outputs agreed in the sixth place in all twelve outputs for every run made. Therefore, the accuracy is exactly the same whether a 16-bit or 32-bit computer is used if floating point numbers are used.

In the past, the main problem in using floating point arithmetic has been that the subroutines are slow and floating point hardware doubles the cost of the computer. Therefore, the timing of the compact program was measured under worst case conditions to see if real-time computation was a possibility. The compact program was run through eighty complete cycles in slightly under fifteen seconds. This represents 40 seconds of real-time when the integration interval is one-half second and 80 seconds when H is one second. The duty cycle is 38% in the first case and 19% in the second. This allows plenty of time for additional system simulation, wave generation, and input/output subroutines.

The core size presented a problem. The basic core size for a small computer is 4096 words and any above this amount requires the core to be extended to 8192 words, thus increasing the cost. An analysis of the core used for the compact program is given in table 16.

The simulation program needs 2632 words without I/O and system simulation. The FORTRAN I/O routines were were used as an indication of what was needed and the total was just over the 4096 limit without including wave generation and tank simulation. It seems unlikely that enough cutting could be done to get the entire program under this amount, and therefore an 8192 core is indicated.

TABLE 16. SUBROUTINE CORE SIZE

Subroutine	Size	Total
UPDATE	1253 words	
SQRT, SIN/COS	317	
FLOATING POINT	664	
DATA TRANSFER	130	
COMMON	268	
	2632	2632
MAIN	417 words	
FORTRAN I/O	1698	
ERROR CHECK	215	
	2330	2330
		4962 words

Hand coding might help, but then all of the advantages of making changes easily, not having to worry about scaling, and being able to work with the simulation program without having to learn a particular language would be lost.

Fixed point hardware multiply-divide might help the problem. The compact program was run on a SDS Sigma 5 with this option and the core space was cut in half. Hardware M/D is about one-third as expensive as extending the core.

Floating point hardware doubles the cost of the computer and is used only when speed requirements make it necessary. It cannot be justified at present, but the cost keeps coming down as more computers come onto the market.

A further cost is a real-time clock and interrupt. This is necessary to tie the integration interval to the outside world through the I/O system. A teletypewriter is also needed to communicate with the computer. The total costs are estimated below.

Basic Computer	100%
Teletypewriter PP/PR reader	30%
Interrupt	8%
Real-time Clock	8%
Simulator I/O	100%
M/D Hardware	20%
or	
Extended Memory	60%

Therefore, the total hardware cost of a submarine simulator, excluding the motion platform, will run between 285% and 325% of the basic computer.

The I/O equipment doubles the cost of the computer. This section contains the D-A and A-D converter discrete inputs and circuits that

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tie the computer to the motion platform. Generally, it is designed for the particular job and this is the reason for the expense.

There are two ways in which this cost can be reduced. One is to do all programing at a central location. A computer is chosen whose manufacturer has software that simulates the program assembly on a much larger computer. A program is written in FORTRAN and run on a Sigma 7, for example. A paper tape is produced that contains an object program for the small machine. This tape is sent into the field and loaded with a simple tape reader at the user's facility. A good program will write the most compact program possible using the advantages of the large computer. This saves the cost of the teletypewriter for each small computer. More importantly, it reduces the cost of human programing.

The second step uses a read-only memory in the small machine. The program is written, debugged, and tested at a central facility. A tape containing the complete program is sent to the computer manufacturer who returns read-only memories containing this program for each simulator. These are plugged into the small computer at each location and no programing or testing is required at all. This could be the most economical method of all if a number of simulators were going to be produced.

A review was made of the small computers now on the market. Up to a year ago, the only computers on the market were 16-bit machines in the class of the SEL810A (used in the 20A62 Emergency Ship Handling Trainer), the PDP-8 and DDP-516. These machines ran from $18,000 up. Since then, a number of different manufacturers have come onto the scene and prices are dropping while speed is increasing. The list below includes some of the computers that should be considered and represents the state-of-the-art at the time of this report. It cannot be complete because of the rate at which the field is changing. Price includes 4K of memory and a teletypewriter.

TABLE 17. SMALL COMPUTER DATA

Computer	Manufacturer	Price
Data 620i	Varian Data Machines	$14,500
DataMate 8	Gamo Industries	$15,000*
HP 2114A	Hewlett-Packard	$12,000
Interdata 3	Interdata	$11,200
NOVA	Data General	$10,000
MAC 16	Lockheed Electronics	$12,000

*price includes 8K core

It would be advisable to run the compact simulation program on a particular computer before the final decision is made.

G. **VERIFICATION**

Section C-1 of Specification 341-101, Study of Submarine System

Equations, requires that certain tests be made to verify that programs having reduced sets of coefficients are compatible with the full set of coefficients as determined by David Taylor Model Basin. These tests shall be computer runs simulating submarine operation through a varied set of controlled maneuvers such as meander, overshoot, etc. Tests will be run for the full set of coefficients and each different set of reduced coefficients.

Three groups of runs were performed; one for the full set of coefficients, one for the modified set, and one for the basic set. The modified set had twenty-one coefficients from the full set to zero, and two other coefficients changed to help moderate the effects of eliminating coefficients. The basic set has twenty-four coefficients set to zero and four other coefficients changed to help offset the effects of eliminating coefficients.

The submarine maneuvers that were performed by computer to verify the compatibility of the reduced sets are described below.

1. MEANDER

This maneuver demonstrates the capability of a submarine to recover from a rise or a dive without use of primary control surfaces or emergency ballast changes. It provides an evaluation of the longitudinal stability of the submarine.

The submarine is operated at a given speed on a given course until steady, level flight is achieved. The submarine is then disturbed from its neutral pitch angle by deflecting the sternplanes a fixed amount in either rise or dive. After the submarine reaches a preselected pitch angle called execute pitch angle (Θ'), the planes are returned to their original, neutral position. They are held fixed while the submarine is allowed to pull out and resume a condition of steady, level flight. The resulting pitch angle trajectory provides both visual and quantitative evaluations of the longitudinal stability of the submarine.

Meander tests were run on the digital computer, program EB920, Submarine Simulation Program, for each set of coefficients at five different command speeds (5, 10, 15, 20, and 25 knots). Maximum diving plane angle was 20 degrees, and the execute or pull out angle was 10 degrees on all meander maneuvers.

Computer-plotted graphs of sternplane angle versus time, pitch angle versus time, and depth versus time were recorded for each run. The damping factor was calculated for the full set data and modified set data in accordance with methods detailed in BuAer report, "Dynamics of the Airframe".

The curves for the basic data showed greater damping and could not be calculated by the same method. Therefore, the damping ratio was estimated by comparison of these curves to other sets and prior general knowledge of the relation of damping factor to response curves.

2. VERTICAL OVERSHOOT

The overshoot is a specific maneuver utilized to provide numerical values of the inherent effectiveness of the diving planes in initiating and checking changes in depth.

In this maneuver, the submarine is operated at a fixed command speed and course throughout. The controls are maintained for level flight until the selected speed is achieved. The stern elevators are then deflected to a given angle and are held until the submarine reaches preselected pitch angle, called execute pitch angle (Θ'). At that time, the elevators are reversed an equal and opposite amount from the original neutral position and held until the rate of change of depth reverses.

This maneuver was performed on the digital computer for the three sets of coefficients, at five different command speeds (5, 10, 15, 20, and 25 knots). Maximum diving plane angles of 10 and 20 degrees were also used for two complete groups of curves. Execute pitch angles of 4, 8, 12, and 16 degrees were also selected.

Computer-plotted graphs of stern elevator position, submarine pitch angle, and depth versus time were prepared for all runs. Units given are radians, feet, and time.

From the data of these runs, a number of graphs of parametric curves were prepared and machine-plotted which provide relative measures for the runs using the three different sets of coefficients.

3. STEADY, SUBMERGED TURN

This is a test to investigate the submarine response to action of its rudder in horizontal plane maneuvers; in particular, loss of speed, steady turning diameter, roll angles, and time to reach 90- and 180-degree change of heading.

In this maneuver, the submarine is operated at a fixed command speed throughout. The computer program provides automatic pilot operation during this maneuver to maintain depth as constant as possible. The controls are maintained fixed until steady, level flight at the selected speed is achieved. The rudder is then deflected to a given angle and held steady. During the maneuver, computer-plotted graphs were made of rudder angle (DR), sternplane angle (DS), pitch angle (Θ), roll angle(ϕ), heading angle (Ψ), normal velocity (w), yaw angular velocity (r), forward speed (u), and depth (z). The angle of attack (α) was described by this equation for small values of w.

$$\alpha = \tan^{-1} \frac{w}{u} \approx \frac{w}{u} \ (\text{for } \alpha < 10°)$$

Therefore, the plotted values of w may be used for values of α.

The data from these runs was used to prepare graphs of parametric curves which provide relative measures for the runs of full set coefficients and runs of modified set coefficients. Results of runs with basic set coefficients were almost identical to runs with the modified coefficients.

4. RPM VERSUS SPEED

Since the thrust coefficients have been neither changed in value nor eliminated, there was no need to repeat these calculations.

5. ACCELERATION/DECELERATION

This test determines the submarine response to change in command speed values.

The autopilot function is operating throughout this test in attempting to hold a constant depth. The submarine is started from an initial at rest or zero velocity condition at 800 feet depth. The command speed is increased in 5 knot increments up to a maximum of 25 knots at equal time intervals of 230 seconds. The command speed is then reduced in 5 knot increments to zero over the same time intervals.

Runs were made for full, modified, and basic sets of coefficients.

6. MAXIMUM ACCELERATION/DECELERATION

This test is an examination of the submarine response to a large change in command speed or throttle setting.

The autopilot is in operation during this complete maneuver. The submarine is maintained in steady, level position at 800 feet depth. The command speed is then immediately increased to 25 knots. The value is held for 500 seconds and then abruptly reduced to zero.

During this maneuver, computer-plotted graphs of the same parameters as used in Section 5 were made versus time, S_s, Θ, u, w, q, and z.

Runs were made for full, modified, and basic sets of coefficients.

7. LONGITUDINAL TRIM

This test shows the submarine response to a small change in elevator deflection.

The submarine is held at level flight at a particular speed and depth, the sternplane is then deflected a small amount, and the changes in certain dynamic parameters are noted. A charge of 3 degrees was used in this case. These runs were performed at 5, 10, 15, 20, and 25 knots for all three sets of coefficients over a time period of 240 seconds.

SECTION IV

RESULTS

A. RESEARCH PROGRAM RESULTS

The research program simulates the equations of motion as expressed in NSRDC Report 2510[1]. The use of this program was described in the section on "Methods and Procedures" of this report. A brief description follows.

EB920 - Submarine Simulation - This program calculated the dynamic changes in submarine position, velocity, and attitude as a function of time in six-degrees-of-freedom. It consists of the following parts.

Main Program - This program ties the subroutines together, calculates a few constants, initializes all arrays, writes out the hydrodynamic coefficients used, writes out the twelve rates and positions as the program proceeds, writes the plotting tape if plot subroutines are available, and finally checks to see if multiple runs are required.

Subroutine INPUT - Reads in a deck of cards with all coefficients and initial values, stores values for initializing during multiple runs.

Subroutine KUTTA - Integrates twelve variables with a Runge-Kutta 4-step integration subroutine.

Subroutine INTEG - Does the same job as KUTTA, but has a choice of three simpler integration routines.

Subroutine EVAL - This routine simulates the actual equations of motion for integration with the KUTTA subroutine.

Subroutine EVAL 1 - Same as EVAL except uses subroutine INTEG.

Subroutine CONTR - This subroutine allows the researcher to program an input which follows standardized submarine tests for climbing turns, flat turns, meanders, overshoots, and impulse inputs.

Subroutine INVER2 and Subroutine MATMPY - These subroutines invert a matrix and multiply it by a column vector for solving the equations of motion when extreme accuracy is desired.

ZC790 - Submarine Simulation, Longitudinal Freedom - This program is identical to EB920 except that it has only three-degrees-of-freedom. Therefore, only meander, overshoot, and similar runs can be calculated. This program was written to save computer time because it ran in one-third the time of EB920. This will be a real advantage where a large number of longitudinal runs are required. It is limited to 2nd order Adam's integration, so less core space is required.

A number of additional programs were written to assist the researcher in submarine simulation.

EC430 - This program uses the location and weight of all tanks on the submarine to calculate the center of gravity shift limits and weight changes for any ballast arrangement.

EC470 - This program calculates the neutral angles for any speed in level flight. The coefficients are used as an input and the level flight values of plane angles, pitch, and normal velocity are calculated.

ZC300 - This program calculates the amount of thrust and the accelerations of the submarine for the ratio of commanded speed to actual speed.

ZC590 and ZC691 - These programs calculate the error present when values from two different runs are supplied according to the criteria set up to evaluate the effects of changes.

The next group of programs are used to determine the exact natural frequency and damping ratio of either the longitudinal or lateral channel. An impulse is used to excite the simulation program and the values of pitch angle or bank angle are punched on cards. These cards and the solution to the linear characteristic equation are used to determine the numerical values of the time response equation.

EC140 and EC200 - These programs solve for the roots of the characteristic equation of the system in longitudinal and lateral channels respectively.

EC150 and EC330 - These programs use the output of the above programs and the time response cards to solve for the coefficients of the time response equation.

EC310 - This program uses the output of EC150 or EC300 as initial conditions to solve for a "least squares fit" against the submarine solution output cards. The fitted and actual response is either listed or plotted for comparison purposes.

The above programs will allow further research in the field of submarine simulation on a general purpose digital computer equipped with a FORTRAN compiler. The final program is complete, flexible, and can be used for both research and check-out.

One of the most important features is that a complete test guide can be run for any set of coefficients before the simulator is built. Any approximation can be checked in a week's time against the full set of coefficients as supplied by NSRDC. If plotting subroutines are available, graphs can be made of any output parameter using standard submarine test inputs as a reference.

B. NSRDC COEFFICIENT RELIABILITY

The equations of motion can represent a particular submarine only if the coefficients supplied by NSRDC truly represent that submarine. For the demonstration submarine, these coefficients did an excellent job of simulating the correct output, in spite of some differences between the various NSRDC reports supporting these coefficients.

C. NSRDC EQUATION ANALYSIS

1. COEFFICIENTS - MODIFIED EQUATIONS

The standard equations of motion were analyzed using a demonstration submarine as an example. Coefficients were divided into longitudinal, lateral, and cross-coupling types and were analyzed as to importance. A submarine trainer that will simulate a particular submarine can use the following approximations. Keep the coefficients in table 18.

TABLE 18. FINAL NON-ZERO COEFFICIENTS

Axial	Lateral	Normal	Roll	Pitch	Yaw
XUD	YPD	ZQD	KRD	MQD	NRD
XDSDS	YRD	ZWD	KPD	MWD	NPD
XDBDB	YVD	ZQ	KVD	MQ	NVD
XDRDR	YR	ZW	KP	MW	NR
A11	YP	ZSTR	KV	MSTR	NP
A12	YV	ZDS		MAWQ	NV
A13	YDR	ZDB		MWAW	NVAV
A21	YVAV	ZWAW		MDS	NPQ
A22		ZVV		MDB	
A23		ZRR		MRR	
A31		ZVR		MVR	
A32				MVV	
A33				MRP	

IX, IY, IZ, ZB, XG, ZG   NDR is reduced by 20%, KVAV is reduced by 50%

The coefficients in table 19 are set to zero.

TABLE 19. FINAL ZERO COEFFICIENTS

Axial	Lateral	Normal	Roll	Pitch	Yaw
XQQ	YPAP	ZPP	KQR	MPP	NQR
XRR	YPQ	ZRP	KPQ	MQAQ	NRAR
XRP	YQR	ZAQDS	KPAP	MVP	NWR
XUU	YVQ	ZWAQ	KR	MAQDS	NWP
XVV	YWR	ZAW	KVQ	MAW	NVQ
XWW	YARDR	ZWW	KWP	MWW	NARDR
XVVE	YVAR YWP	ZQE	KWR	MQE	NAVR

TABLE 19. FINAL ZERO COEFFICIENTS (cont.)

Axial	Lateral	Normal	Roll	Pitch	Yaw
XWWE	YSTR	ZWE	KSTR	MWE	NSTR
XDRDRE	YVW	ZWAWE	KVW	MWAWE	NVW
XDSDSE	YRE	ZDSE	KDR	MDSE	NRE
XWQ	YVE	ZVP	KSTRE		NVE
XVR	YVAVE				NVAVE
	YDRE				NDRE

IXY, IXZ, IYZ, XB, YB, YG

2. COEFFICIENTS - BASIC EQUATIONS

The coefficient list and programing for a simplified basic trainer
is identical to that above except for the following changes.

> ZWAW, MWAW, MAWQ are set to zero.
> ZG is set to a constant value
> ZW is raised by 75%
> MW is lowered by 11%
> $\sin\Theta = .971\Theta$
> $\cos\Theta = 1.02 - .281\,\Theta$
> $\sin\phi = .971\phi$
> $\cos\phi = 1.02 - .281\,\phi$
> X and Y are not computed, so ψ is not necessary.
> Therefore, $\dot\psi$ is not integrated.

Actually, eliminating ZWAW, MWAW, and MAWQ results in such a small
reduction in the total program complexity that it would seem advisable
to keep them as in the modified program, unless such a reduction would
allow a smaller core.

3. KINEMATIC RELATIONS

The Euler angles should be computed in the following order.

$$\dot\Theta = q \cos\phi - r \sin\phi$$

$$\dot\psi = \frac{q \sin\phi + r \cos\phi}{\cos\Theta}$$

$$\dot\phi = p + \dot\psi \sin\Theta$$

4. INTEGRATION METHODS AND TIMING

The best integration method for submarine simulators is 2nd order
Adams which has the difference equation given below.

$$y_n = y_{n-1} + \frac{h}{2}\left(3\,\dot y_{n-1} - \dot y_{n-2}\right)$$

It is both simple and stable over the entire operating range of computation. The value of the integration parameter H should be 0.5 second for an accurate modified trainer and 1 second for a basic maneuvering trainer.

5. NEAR-SURFACE COEFFICIENTS

The use of near-surface coefficients in a submarine simulator is not necessary. For particularly accurate operation close to the surface, MSTR and ZSTR should be computed as a function of depth and speed, but keeping the deep coefficients otherwise does not change the near-surface handling qualities to any great degree.

A hydrodynamic analysis was made which resulted in the determination of the magnitude of the coefficients used for wave effects on the demonstration submarine.

D. MODIFIED AND BASIC PROGRAMING

One program was written for both the modified and basic programs. This program was written in two parts. The first part takes the data deck containing all coefficients used with the full simulation program, and produces another data deck containing the reduced coefficients. This reduction is done on the same computer as the one used for the research program.

The second part is the actual simulation program. It can be compiled on any small general-purpose digital computer that has 8K of core and a basic FORTRAN compiler. Usually, the latter requires the former. The input to this program is the CONTR subroutine and the output is twelve variables in a teletypewriter format for demonstration purposes. If the proper I/O subroutines and channels were available, this program could be run in real-time.

E. WAVE GENERATION

An investigation was made into the simulation of waves in a digital computer. Waves are treated on a statistical basis with a particular power spectra for each sea state. A literature search was made that resulted in two different mathematical models and a number of different spectra in current use. A bibliography is given on the current state-of-the-art.

A particular spectra was chosen and two math models were developed. Digital programing techniques were applied to these two models and a sample sea state was produced and checked by means of a power spectral density run.

F. SMALL COMPUTER UTILIZATION

A small general-purpose 16-bit computer can be used in real-time for the submarine simulation problem. The computer would have to have

8K of core and an add time of approximately 2μ sec. There are at least six low-cost computers now on the market in this class.

G.   VERIFICATION

Finally, a complete verification program was performed in which the full range of submarine maneuvering operations was covered for the basic, modified, and full-set coefficients. They were in very close agreement, with the modified set in particular having almost no differences from the full set.

SECTION V

DISCUSSION

A. HISTORY AND REVIEW OF THE STATE-OF-THE-ART

The art of submarine simulation is older than that of aircraft simulation. This is a natural consequence of the shipbuilding industry which has used models in towing tanks for hundreds of years. The fact that ship performance could be expressed by a mathematical model has been a well-known fact to the theoretical workers in the field ever since Froude and Reynolds worked out the details in mid-nineteenth century.

During WW II, the science of feedback control theory caught up with the mathematical structure that had been built over the years. This resulted in the Askania diving trainer of which four are still in use at the Submarine School in New London, Conn. The computer is a mechanical one that uses compressed air, and the servoes are controlled by a jet pipe unit. This is a type of feedback controller that consists of two jets of air blowing on each side of a flapper vane. One is the controller and the other is the follow-up. This idea has come back into vogue in today's fluidic amplifiers.

These trainers were followed in the early 1950's by the 21B20 built by the Electric Boat Co. They started out with "Lamb's Hydrodynamics" and a paper by the Society of Naval Architects and Marine Engineers called "Nomenclature for Treating the Motion of a Submerged Body Through a Fluid", April, 1950, and ended up with an analog computer that had 210 amplifiers and 41 multipliers. The trainer simulated four different classes of submarines with two different motion platforms. However, only one platform could be used at a time; and to change either platforms or classes took a full day and included changing cables, panels, and summing junctions. This trainer was later updated to the 21B20A.

The 21B20A was followed by the 21B56, which was a Basic High Speed Control Trainer. The "Training Effectiveness as a Function of Trainer Complexity" study by J. Newton, Electric Boat Division, Groton, Conn., was conducted between the delivery of these two devices. Device 21B56 was the first Navy trainer to reflect the findings of this study. The computer size was reduced by a factor of ten. The simulation in this trainer was simplified to the degree that all that can be said for it is that the indicators move when the controls are pushed.

This 21B56 was updated to the 21B56A with the addition of a ballast control panel. The weight and moment computations were added to the hydrodynamic computer and a complete tank simulation was designed. The final result was that about $2\frac{1}{2}$ times as many amplifiers were added to increase the simulation capability.

These simplified equations of motion were also used in the 21A37 Fleet Ballistic Missile Trainer. This is the largest training device

at New London and the calculations for the position and velocity of the submarine form a very small part of the total simulation.

In spite of the fact that submarine maneuvering forms a small part of the training usefulness of the 21A37, it was thought to be necessary to upgrade the flight system with the addition of casualty flooding. The upgraded version was the 21A37/3. The main changes were in the simulation for ballast tanks, engine power for backing down, and calculation of the pitch trigonometric functions. The result was a more complex but more realistic simulation.

In 1967 Hydrosystems, Inc., replaced the analog computer associated with the 21B20A with a digital computer. This was followed by the 21C5 trainer which was a digital trainer from the start. The 21B20A update uses a DDP124 computer with a 16K 24-bit core. At the present time, it is running both platforms at once on a side-by-side basis in the same computer. An interesting contrast can be made between analog and digital equipment with this trainer. The DDP124 computer sits in the middle of a very large room that was once filled with analog equipment. The programing of this trainer includes all of the NSRDC coefficients.

The point of the above history is that the simulation of submarines started out on a very complex level. As a reaction to this, the next generation of simulators was too simple and had to be upgraded. The simulation is beginning to move back to the complex side and it is hoped that this Submarine Equations Study will strike a happy medium.

There have been as many sets of equations of motion as there have been submarine trainers. However, a close examination of the dynamics reports seems to indicate that they are pretty much the same except for the degree of complexity felt necessary by the system analyst who actually programs the simulator.

One difference that did come to light was two schools of thought as to how an equation should be normalized. The coefficients of the equations of motion are reduced to non-dimensional form by dividing through the calculated value by the fundamental units necessary to remove any dimension. For example,

$$\dot{u} - K \delta_s^2$$

where $\dot{u}$ is forward acceleration and δ_s is the sternplanes. This is reduced to

$$m\dot{u} = \frac{\rho}{2} \ell^2 u^2 X_{\delta_s \delta_s} \delta_s^2$$

If the dimensions are checked on both sides of this new equation with δ_s in radians, it will be seen that $X_{\delta_s \delta_s}$ is dimensionless. The difficulty is with ℓ^2 term. This is the length of the submarine squared. Some authorities use the Characteristic Area "A" which is the cross-sectional area of the submarine and which has the same dimensions as ℓ^2 . This procedure comes from "Nomenclature for Treating the Motion of a Submerged Body Through a Fluid", SNAME Technical and Research Bulletin No. 1-5 and is perfectly correct if the coefficients were reduced using this parameter. The difficulty occurs when the reduction is made using ℓ^2 and the simulator recalculates the original value using "A" without adjusting for the differences in the two characteristics.

It is this type of conflict that the publication of the Naval Ship Research and Development Center publication "Standard Equations of Motion for Submarine Simulation", Report 2510 will help eliminate. For the first time, a set of equations of motion are available that can be considered as a standard, and this should ease the burden on the system designer to a great degree.

B. DETERMINATION OF SIMULATION REQUIREMENTS

In order to determine the simulation requirements for a submarine simulator, background material that would be useful to a trainer designer must be collected. Investigation includes a review of the state-of-the-art, interviews with submarine personnel and trainer users, establishment of data sources, determination of the validity of the data establishing the desired ranges and accuracy requirements, and reduction of this mass of material into performance requirements that can be used to determine the simplification to be applied to the equations of motion. In many cases, the information gained cannot be applied directly to establishing a performance characteristic but must be used when making a judgment of the final output.

The first source of information was interviews with submarine trainer users. They could be divided into two classes; namely, those who were submariners and those who were interested in the utilization of submarine trainers. These interviews could be very frustrating because many men who have served on submarines know what happens during a particular maneuver, but they cannot remember how it happens. For example, a snap-roll occurs at the start of a turn, but it was difficult to find out if this occurred to the outside or the inside of the turn. Apparently, different classes of submarines can do either. In general, the men in charge of training at the Submarine School were the most knowledgeable and could explain what should happen.

The consensus of opinion was that the Navy is satisfied with the training values of the simulators now on hand, but that some of them are too simple for complete training. One interesting comment was that one of the trainers equipped with the full set of NSRDC coefficients was the best simulator compared to the actual submarine that they had used. However, another trainer using the same equations of motion was not as good and did not represent a particular submarine as well as the

the other. This shows that the NSRDC coefficients and equation of motion can do an excellent job over the full operating range, but it is possible to have a wrong value get in along the way.

Control in the longitudinal channel should be very precise. When a 300 foot submarine dives at a 35 degree angle, which is not unusual, there is a 180 foot difference in depth between the bow and stern. Therefore, in shallow water, extreme care has to be taken when changing depth. Most maneuvering training involves depth changes and a good planesman can come onto a preordered depth with no overshoot at all. Even a few feet over is sloppy handling. Changes in course are not as much of a problem, so simulation can be considerably more lenient in this area. Therefore, the philosophy was to simulate the longitudinal channel as closely as possible, but to allow considerable freedom on the response of the lateral channel as long as the magnitudes were simulated to an approximate degree.

The interviews disclosed that there were a number of real-world effects that did not transfer into simulator training through the equations of motion. This included such things as the tendency on the part of one submarine to dive when making turns to the left but not to the right, and the variances between different submarines of the same class. However, these do not have to be simulated as long as the overall response of the trainer seems to be valid to the trainee.

Until the present time, submarine maneuvering trainers were mainly used for training at deep submergence, but near-surface operation is becoming more and more important. Generally the older trainers had a simple sine wave generator for surface effects, but this is not good enough any more. Sine wave generators are limited because of their periodic nature. A statistical wave generator is needed so that an occasional wave will be much larger than the others. This is to give training to prevent the submarine from broaching when at periscope depth, which is an unpardonable sin to a submarine captain. The Submarine School has only one simulator with such a wave generator in it, and not enough training has been given in order to determine how effective it is.

One other factor was emphasized in discussing the use of submarine simulators. The purpose of a maneuvering trainer is different from that of an aircraft simulator. In an aircraft, the pilot, or pilots, is the only one who is taking action to control the vehicle. The purpose of the trainer is to have the pilot respond instinctively because the situation can change faster than there is time to think about it. On the other hand, a submarine trainer attempts to train the officers to react in a team effort. At least five to seven people are required to control the path of the submarine and each one has to work as part of the team under any conditions they may encounter. As the amount of reserve buoyancy on the newer submarines is decreased, the actions by an individual become more and more critical while the amount of time to respond in an emergency becomes less. Since the required response to the same casualty in two different classes can be exactly the opposite, the simulation fidelity in an advanced trainer has to be valid even at

the extreme ends of the operating range of the submarine.

The second source for information to determine the simulation requirements for a trainer that simulates a particular submarine is the literature available on that submarine. The user of this study should be thoroughly familiar with the NSRDC publications and the handbooks that are available. This is covered in part D of this section. They not only cover the numerical values for the submarine, but they also give a physical description of any unusual maneuvers that might be encountered and that would need consideration for inclusion in the final simulation.

A problem that has always been associated with any simulator is that of the range and accuracy of the variables in the equations of motion. This is because an analog computer has a certain definite resolution and noise level based on the computing voltage used for full-scale of any variable. This is especially true under conditions of level flight because many of the variables, and thus the voltage, are close to zero and the computer is trying to close the loop and compute response to a signal that is equivalent to the noise present. If the range of a variable is reduced to improve the resolution, then saturation may occur when normal maneuvers are undertaken.

This is not a problem in a digital computer, since only numbers are being operated on and the actions of the machine are exactly repeatable. The accuracy and resolution of the machine is limited by the number of bits in the computer word. This is usually twenty-four, even in a sixteen-bit computer where two words are used, and so a change in the least significant bit is equal to 6×10^{-8} part of the total resolution of each variable. Therefore, a continuous error of one bit at each integration cycle at one second real-time intervals in computing an addition would result in one-tenth of one percent error over fifteen minutes operation. Range is separate from resolution in floating point notation because an exponent is computed along with the fractional part of the word. This can be up to 140 orders of magnitude even in a small computer.

The actual accuracy of the system as a whole is limited by the I/O equipment which was not covered as part of this study. The output conversion should be as accurate as the trainees' instruments and as accurate as the motion platform feedback pick-ups. The input accuracy should be good enough so that an accurate conversion is made on the order of one-tenth of one percent when the controls are being moved at their fastest rate by the operator.

The range of all variables would have to be known if fixed-point coding were used. Casualty runs would have to be made using the supplied program to determine the maximum ranges under all conditions. This is not necessary if FORTRAN programing is used.

C. VERIFICATION REQUIREMENTS

There is a need for verification at three different points in the

investigation of a set of coefficients to be used in a submarine simulator.
It should be determined that the coefficients match the submarine being
simulated, that individual changes made in the coefficients do not affect
the simulation beyond a certain degree, and that the complete reduced
simulation does the same job as effectively as the full set of coefficients.
The handling qualities of the simulator should be the same as the submarine,
but the output to a particular set of inputs is what is actually measured.
Because the research program is limited to non-real time, measurements
over a wide spectrum of operation have to serve in place of pilot evaluation
which is usually used.

Discussion of the methods of satisfying the first two requirements
have been covered in the sections of this study on impulse testing and
reduction of coefficients. The procedures of the third requirement have
been covered in the section on Verification under Methods and Procedures.
The results are given in Advanced Submarine Systems Equations, NTDC
Report Number 68-C-0050-2.

Generally, a concise definition of how two sets of outputs are to
be compared cannot be written until the outputs have been generated.
However, some rules were followed that apply to most of the verification
runs that were made. These were a result of discussions with Goodyear
Aerospace's Human Factors Department. It was decided not to apply perfect
phase criteria to the comparison plots. This is because the man in the
loop cannot determine the passage of time to a precise degree. Phase
differences show up either as a lead or lag on the response plots. What
is important is that steady-state values remain the same and that the
magnitude of easily recognizable points be the same. For example, the
snap-roll angle at the start of a turn should be simulated accurately.
In some cases, the comparison curves have no definite points to compare.
This is generally true when the damping ratio is high, and in these cases
the response was compared at the end of two minutes. The difference was
held to five percent change of the change between that point and the value
at t=0. The five percent figure was considered as below the threshold of
observation on the part of the planesman, and the two minutes is a human
factors estimate of how long an operator in a closed loop can remember
the past actions.

A number of tests were run as part of the major verification pro-
cedure incorporated into the contract for this study. Volumes III and
IV contain only the output as a result of these tests and the total
number of graphs produced was over 550. As it turned out, this extensive
testing proved to be of benefit to the study because it showed up several
weaknesses in the coefficient reduction that was used in the first mathe-
matical model. However, for a test guide to be used with a simulator,
the output would not have to be nearly as extensive as was required in
this case. The various submarine tests are discussed from the point of
view of producing a test guide for a training simulator in the field.
The tests themselves are described in the section on Verifications.

Meander - In this case, only sternplane angle and pitch should be
recorded. Depth will be measured during the overshoot runs. Instead of

calculating the damping ratio, the criterion should be to match the amplitude peaks to within five percent of the amplitude of the first peak and the zero crossings should be within five seconds of the values of the standard. The five different speeds are a suitable number, but one of them should be at the point of maximum damping ratio, which is approximately 7 knots in the demonstration submarine.

Vertical Overshoot - These tests determine the handling qualities in the longitudinal channel. They required forty runs to cover the range of operation. If the NSRDC coefficients are being used, a valid check can be made at one high speed and one low speed. The output seems to be linear with the driving plane angle, so one angle should be sufficient. Pitch execute angles should be five and twenty degrees. This reduces the verification tests to four, or one-tenth the number above. The only time that a great number of overshoot tests should be performed is when a guide is to be prepared for training use. The planesman does not know how big a change is required for any depth change, particularly on the basic trainer which does not represent any specific submarine. If the operator is supplied with a chart of "Overshoot Pitch Angle versus Approach Speed" and "Total Depth Change versus Maximum Pitch Angle" for the basic trainer, he can predict what plane angle changes are required for a specified depth change. However, for test guide purposes, the four curves should be enough if they are matched on the same basis as the meander curves.

Steady Submerged Turns - These verification tests required that eleven parameters be recorded at three rudder angles and five different speeds for a total of 165 charts. Much of this information is not used by the system programer to check the operation of the simulator and the following is suggested. The rudder angles should be 15° and 35° and the speeds should be 5 and 25 knots. The parameters recorded as a function of time are: rudder angle, sternplane angle, speed, heading, roll, and depth. Another requirement is that these turns should be made under autopilot control, using the sternplanes to close the loop in keeping a constant depth. The point checks are snap-roll angle, steady-state roll angle, change in speed at steady-state conditions, and heading change at the end of a specified period of time.

The reason for using autopilot control and recording sternplane angle and depth is that although these do not change very much during a steady-state turn, their shape over time is very important for checking all six axes in the simulation. Open loop control will not serve because the longitudinal channel is frozen and the effects of lateral motions on elevation is not apparent. These effects can be quite large with a small change in some of the coefficients.

Acceleration/Deceleration - Two tests are given in this study, although only one was required. The test given in the specifications requires several minutes of computer time to run and it is felt that the maximum acceleration/deceleration test will verify the same operation in a shorter time. The submarine is at zero speed and a command speed of maximum is called for. When this speed is reached, a command speed of zero is called and the program runs for the same length of time again.

The curve of speed versus time should match to a close degree to insure good thrust simulation.

Longitudinal Trim - This tests for conditions that should be verified under meander and overshoot.

In this discussion, it is assumed that the NSRDC coefficients are correct, and the approved criteria is determined by the output of the programs produced by this study. In fact, such runs on a general-purpose digital computer should be made part of the specifications for any new submarine trainer. Not only will a test guide be produced, but also a means of producing data will be provided to assist in maintenance and trouble-shooting during the life of the simulator.

D. NSRDC DATA SOURCES

The Naval Ship Research and Development Center (NSRDC) has a complete data package on about 25 different existing submarine designs. Most of this data carries the classification of "CONFIDENTIAL" so that before it can be furnished to an outside contractor the proper clearances and need-to-know has to be established through normal channels. Once this is done, a complete set of data can be requested from NSRDC. The programs developed in this study use the NSRDC mathematical model as a base, and therefore such a data package is essential before simulating any particular submarine. This section describes what is available in the set using the demonstration submarine as an example. The name and number has been replaced with NAME and # in the titles of reports. The titles are the same except for this so any report can be requested by inserting the proper name and number in the request to NSRDC or the Naval Training Device Center Technical Library at Orlando, Florida. They can also be received through the Defense Documentation Center in Virginia with the necessary clearances.

The essential report for the simulation of the equations of motion is the Naval Ship Research and Development Center Report 2610: "Standard Equations of Motion for Submarine Simulation". This unclassified report, which was described above, contains the equations of motion directly applicable to submarine simulation and some description of their background, how to use them, and how data is generated in order to determine the coefficients for a particular submarine.

The equations of motion listed in this report are applicable to the rigid body motions of any submarine or submerged vehicle. They are written to use non-dimensional coefficients, and it is these coefficients that are needed to simulate realistically a specific submarine. A complete set of input data consists of the dimensions of the submarine being simulated, the values of the inertia terms, and values for the hydrodynamic coefficients.

The numerical values can be requested from NSRDC under the title "Hydrodynamic Coefficients and Other Terms for Equations of Motion for #(U)". They consist of three pages of preprinted forms containing spaces

for all the coefficients listed in NSRDC Report 2510 along with the dimensions, buoyancy and inertial terms. They do not contain values for what is called systems simulation in a complete submarine simulator. The addition terms needed if a real-time closed loop simulation is to be attempted are such values as thrust time delays, control surface rates, and time histories of the ballast tanks during flowing and venting. With this list of numerical values and the equations in Report 2510, it is possible to simulate any time outputs, as a result of any specified input conditions, on a digital computer.

The following reports are not needed if the equations were programed without error and the coefficients are correct and accurately represent the submarine being simulated. Since this is usually not the case, it is necessary to have information against a set of input conditions. Hydrodynamic coefficients can be calculated generally from two different sources: towing tank tests and full scale sea trials. A report generally covers each of these areas.

"A Full-Scale Evaluation of the Handling Qualities of the NAME Class Submarine (#) (U)" gives the results of the full scale sea trails. A number of standard tests have been developed for testing the handling qualities of a submarine. They cover submerged performance in both the vertical plane and the horizontal plane. This report gives the specifications of the submarine, describes the procedures used to conduct the tests in each plane, gives samples of outputs to typical test runs, and summarizes the results over the range of operation of the submarine. Since the input conditions are specified, the full scale tests can be repeated with the computer program and compared with tabulated results to see if the outputs of the simulation are accurate.

"Model Investigation of the Stability and Control Characteristics of NAME (#) at Deep Submergence (U)" contains the results of model testing and a list of the coefficients developed from these tests. They should be the same as the list obtained from NSRDC except for the fact that work is always being performed to refine the numerical values. This report approaches the subject of response on a frequency basis rather than on a time basis as in the full scale trails. The outputs of the model tests are used as solutions to a set of differential equations and the various roots of the characteristic equation are used to determine the coefficients. These roots can be used to determine the characteristic frequency and damping ratios as a function of speed. The simulation program produces the same information in response to impulse testing and thus a comparison can be made. This report also discusses control effectiveness, neutral angles and critical speed. It is essential that these be correct for the simulation because they determine steady-state operation and the trainee will notice errors in this area before anything else.

The numerical results of the coefficients given by the two above can be compared in "Full Scale Evaluation of Static Stability and Control Derivatives of NAME (#) (U)". This report gives plots of the model curve against the full scale data points.

If any work is to be performed in the area of near-surface operation, the report "Simulator Studies of Depthkeeping Ability of the NAME (#) Under a Seaway (U)" is helpful. This report contains the changes in the coefficients necessary to actually simulate operation under a free surface, and submarine responses to waves while trying to hold an ordered depth. This publication also contains the constants for a NSRDC autopilot that proved helpful to the present study.

"Submerged Turning and Maneuvering Characteristics of the # Submarine from Free Running Model Tests" supports the figures given in the full scale reports in the horizontal plane. Heading changes, loss of speed, and snap roll angles are given for the model.

"Resistance and Propulsion Characteristics of the (#) Submarine as Predicted from Tests of Model and NAME (#)" and "Machinery Performance Trials (U)" are the model testing and full scale reports respectively that relate the command speed used by the NSRDC equations of motion with the revolution per minute (RPM's) seen by the engines. They can also be used to determine control surface deflection rates.

Finally, there are a number of special reports covering recovery procedures from casualty conditions which vary from submarine to submarine depending on the amount of work that has been done in this area.

The one area that has not been covered in these reports is that of tank simulation. In order to know the shift in center of gravity and weight change due to the ballast tanks, it is necessary to know their location and capacity. This information can be found in the NAVSHIPS "Ship Information Books" and "Training Aid Booklets". They cover hull arrangements, controls systems, piping, electrical systems, fire control and the other systems on the actual submarine.

E. INCORPORATION OF CASUALTIES

Training for casualties is an important part of a maneuvering trainer. This study did not examine response to casualties as such, but provisions were made to incorporate casualties into the computer programing. The information on typical casualties comes from a study performed by Goodyear Aerospace for NTDC titled "Submarine Casualty Control Training".

Typical casualties are ship command and control failures, flooding, and air bank failures.

Ship command and control casualties are those concerned with the diving planes and rudder. Such failures include failure of the planes or rudder either at a fixed position or after moving to the end of their travel. Sternplane angle DS, sailplane angle DB, and rudder angle DR, are located in COMMON of the FORTRAN program. This means that these variables can be controlled from any subroutine that has an identical common statement. At the present time, these variables are scheduled from the CONTR subroutine; but in a submarine trainer, they would be calculated from the Input/Output subroutine. It would be a simple matter to have an

additional input to the I/O subroutine, representing an instructor's failure, to interrupt the normal control channel.

Flooding can be incorporated into the tank simulation system as an additional set of tanks whose fill starting time is controlable by the instructor. The program can be as simple or as complex as desired, but the final results affect the flight section of the programing through the terms shifting the center of gravity and through the mass and weight computation. Again, these variables are located in the COMMON area so that reprograming of the present subroutine is not required.

Air bank failures are incorporated into the normal and emergency main ballast tank blow systems, variable ballast tank systems, and trim and drain systems. Since simulation is not provided for these systems as part of this study, these casualties would have to be programed when the system simulation was done.

F. USES AND LIMITATIONS OF THE SUBMARINE EQUATION STUDY

1. USEFULNESS

The usefulness of this study has been indicated by the topics covered up to this point, but it is worthwhile to restate them here.

a. Coefficient Checks - A set of coefficients can be checked against any available data on the real submarine. If new data is received, further checks can be made easily. No simulation is required.

b. Research in Reduction and Casualties - The programs can be used to investigate the effect of further reductions and casualties. This work could be extended with the aid of a real-time computing system to put the man into the loop in real-time.

c. Test Guides - A complete set of data points can be run off for any simulator for which the coefficients are known. Any range of operation can be reproduced in a very short period of time once the coefficients have been justified. Such data outputs are useful in initial checking of the programing for the simulator and in maintenance and trouble-shooting in the field. Both static and dynamic checks can be made.

d. Compact Programing - When programing is done for a submarine simulator, as many constants as possible are combined into a single term to save on the amount of computation required in the compact mathematical model. These programs will assist in this task by calculating the new coefficients required.

2. LIMITATIONS

The biggest limitation in this study is that operation at the extreme ends of the operating range was not covered. This is usually due to casualties, such as flooding, when attitudes are encountered that would never be entered under normal operation. However, NSRDC states that the

equations of motion include effects due to "backing" on propellers and emergency blowing of ballast tanks even to such extremes as an emergency recovery from a hard-climb sternplane jam, including the buoyant ascent after blowing of ballast. It is felt that the modified equations of motion will respond effectively to these conditions, even though they were not tested in this range, but that the basic set probably would not. There are also a number of subroutines that would have to be written in order to make tests such as emergency recoveries. They are listed below.

a. Tanks - The tank blow system is not simulated. However, the effect of any tank arrangement, not including partially filled tank slosh, can be computed through X_G, Y_G, and Z_G.

b. Controls - The time history for rudder and plane changes are not included except that schedules for all of the standard submarine test maneuvers are provided.

c. Failures - Failures, such as plane jams, are not included. However, many of the hardovers and jams of the planes can be simulated by use of the proper constants in the CONTR subroutine of the main simulation program. Air Bank and flooding failures would also have to be added.

SECTION VI

CONCLUSIONS

The "Study, Advanced Submarine Systems Equations" was a success. The "Standard Equations of Motion for Submarine Simulation" NSRDC Report 2510 was used as a basis for producing a submarine simulation program that could be utilized in a small general-purpose digital computer.

REFERENCES

1. Gertler, M. and Hagen, G., "Standard Equations of Motion for Submarine Simulation, Research and Development Report 2510", Naval Ship Research and Development Center, Washington, D.C., June 1967.

2. Nigro, B.J., "Study of Numerical Integration Techniques for Real-Time Digital Flight Simulation", Report #AMRC-TR-67-4, Bell Aerosystems for Aerospace Medical Research Lab, WPAFB, March 1967.

3. Jennings, W., "Introduction to Numerical Methods", Macmillan, New York, 1964.

4. Conrad, K.L. et al., "Digital Simulation Techniques Study", GER 12457, Goodyear Aerospace Corporation, Akron, Ohio, Dec. 1965.

5. Nigro, B.J., Ibid.

6. Baxter, D.C., "Digital Simulation U_ing Approximate Methods", Report MK-15, NCR No. 8630, National Research Council of Canada, July 1965.

7. Thaler, G.J. and Brown, R.G., "Analysis and Design of Feedback Control Systems", McGraw-Hill Inc., New York, 1960, chap.2.

8. Freeberg, C.L., "Introduction to Numerical Analysis", Addison-Wesley, 1965

9. Kaplan and Sargent, "Motion of a Near-Surface Submarine under Waves", Report 63-05, (U), Oceanics, Inc., Dec. 1963.

10. "Final Mathematical Model Report Device 21C5", Report 0061-A001AD, (U), Hydrosystems Inc., Farmingdale, N.Y., Dec. 1967.

11. Cummins, W.E.; Forces and Moments Acting On a Submarine Moving Under Waves - Comparison of Theory with Experiments, Report No. C-596, The David W. Taylor Model Basin, Dec. 1954 CONFIDENTIAL.

12. "Ocean Wave Spectra", Prentice-Hall, Englewood Cliffs, N.J., 1963.

13. Kinsman, B., "Wind Waves", Prentice-Hall, Englewood Cliffs, N.J., 1965.

14. Michel, W.H., Sea Spectra Simplified SNAME, New York, April 1967.

15. Abkowitz, M.A.,et al., Recent Developments in Seakeeping Research and its Application to Design, SNAME, New York, Nov. 1966.

16. Scott, J.R., A Sea Spectrum for Model Tests and Long-Term Ship Prediction, Journal of Ship Research, SNAME, Dec. 1965.

17. Sellars, F.H. and Loukakis, T.A., "The Analysis and Modelling of Irregular Waves", Report 66-5, MIT Dept of Naval Architecture and Marine Engineering, Cambridge, Mass., July 1966.

18. Chen, C.F. et al., "Mathematical Generation of a Realistic Sea", Hydronautics Inc., Technical Report 001-13, Bureau of Ships Contract NObs-78396, Oct. 1963.

NOTATION

Symbol	Dimensionless Form	Definition
a_i, b_i, c_i		**Sets of propeller thrust constants**
B	$B' = \dfrac{B}{\frac{1}{2}\rho \ell^2 U^2}$	Buoyancy force, positive upward
CB		Center of buoyancy of submarine
CG		Center of mass of submarine
I_x	$I_x' = \dfrac{I_x}{\frac{1}{2}\rho \ell^5}$	Moment of inertia of submarine about x axis
I_y	$I_y' = \dfrac{I_y}{\frac{1}{2}\rho \ell^5}$	Moment of inertia of submarine about y axis
I_z	$I_z' = \dfrac{I_z}{\frac{1}{2}\rho \ell^5}$	Moment of inertia of submarine about z axis
I_{xy}	$I_{xy}' = \dfrac{I_{xy}}{\frac{1}{2}\rho \ell^5}$	Product of inertia about xy axis
I_{yz}	$I_{yz}' = \dfrac{I_{yz}}{\frac{1}{2}\rho \ell^5}$	Product of inertia about yz axes
I_{zx}	$I_{zx}' = \dfrac{I_{zx}}{\frac{1}{2}\rho \ell^5}$	Product of inertia about zx axes
K	$K' = \dfrac{K}{\frac{1}{2}\rho \ell^3 U^2}$	Hydrodynamic moment component about x axis (rolling moment)
K_*	$K_*' = \dfrac{K_*}{\frac{1}{2}\rho \ell^3 U^2}$	Rolling moment when body angle (α, β) and control surface angles are zero
$K_{*\eta}$	$K_{*\eta}' = \dfrac{K_{*\eta}}{\frac{1}{2}\rho \ell^3 U^2}$	Coefficient used in representing K_* as a function of $(\eta-1)$
K_p	$K_p' = \dfrac{K_p}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing K as a function of p
$K_{\dot{p}}$	$K_{\dot{p}}' = \dfrac{K_{\dot{p}}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing K as a function of $\dot{p}$
$K_{p\lvert p\rvert}$	$K_{p\lvert p\rvert}' = \dfrac{K_{p\lvert p\rvert}}{\frac{1}{2}\rho \ell^5}$	Second order coefficient used in representing K as a function of p
K_{pq}	$K_{pq}' = \dfrac{K_{pq}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing K as a function of the product pq

K_{qr} $K_{qr}' = \dfrac{K_{qr}}{\frac{1}{2}\rho\ell^6}$ Coefficient used in representing K as a function of the product qr

K_r $K_r' = \dfrac{K_r}{\frac{1}{2}\rho\ell^4 U}$ First order coefficient used in representing K as a function of r

$K_{\dot{r}}$ $K_{\dot{r}}' = \dfrac{K_{\dot{r}}}{\frac{1}{2}\rho\ell^5}$ Coefficient used in representing K as a function of $\dot{r}$

K_v $K_v' = \dfrac{K_v}{\frac{1}{2}\rho\ell^3 U}$ First order coefficient used in representing K as a function of v

$K_{\dot{v}}$ $K_{\dot{v}}' = \dfrac{K_{\dot{v}}}{\frac{1}{2}\rho\ell^4}$ Coefficient used in representing K as a function of $\dot{v}$

$K_{v|v|}$ $K_{v|v|}' = \dfrac{K_{v|v|}}{\frac{1}{2}\rho\ell^3}$ Second order coefficient used in representing K as a function of v

K_{vq} $K_{vq}' = \dfrac{K_{vq}}{\frac{1}{2}\rho\ell^4}$ Coefficient used in representing K as a function of the product vq

K_{vw} $K_{vw}' = \dfrac{K_{vw}}{\frac{1}{2}\rho\ell^3}$ Coefficient used in representing K as a function of the product vw

K_{wp} $K_{wp}' = \dfrac{K_{wp}}{\frac{1}{2}\rho\ell^4}$ Coefficient used in representing K as a function of the product wp

K_{wr} $K_{wr}' = \dfrac{K_{wr}}{\frac{1}{2}\rho\ell^4}$ Coefficient used in representing K as a function of the product wr

$K_{\delta r}$ $K_{\delta r}' = \dfrac{K_{\delta r}}{\frac{1}{2}\rho\ell^3 U^2}$ First order coefficient used in representing K as a function of δ_r

ℓ $\ell' = 1$ Overall length of submarine

m $m' = \dfrac{m}{\frac{1}{2}\rho\ell^3}$ Mass of submarine, including water in free-flooding spaces

M $M' = \dfrac{M}{\frac{1}{2}\rho\ell^3 U^2}$ Hydrodynamic moment component about y axis (pitching moment)

M_* $M_*' = \dfrac{M_*}{\frac{1}{2}\rho\ell^3 U^2}$ Pitching moment when body angles (α, β) and control surface angles are zero

M_{pp} $M_{pp}' = \dfrac{M_{pp}}{\frac{1}{2}\rho\ell^5}$ Second order coefficient used in representing M as a function of p. First order coefficient is zero.

M_q $M_q' = \dfrac{M_q}{\frac{1}{2}\rho\ell^4 U}$ First order coefficient used in representing M as a function of q

$M_{q\eta}$ $M_{q\eta}' = \dfrac{M_{q\eta}}{\frac{1}{2}\rho\ell^4 U}$ First order coefficient used in representing M_q as a function of $(\eta-1)$

$M_{\dot{q}}$ $M_{\dot{q}}' = \dfrac{M_{\dot{q}}}{\frac{1}{2}\rho\ell^5}$ Coefficient used in representing M as a function of $\dot{q}$

$M_{q|q|}$ $M_{q|q|}' = \dfrac{M_{q|q|}}{\frac{1}{2}\rho\ell^6}$ Second order coefficient used in representing M as a function of q

$M_{|q|\delta s}$ $M_{|q|\delta s}' = \dfrac{M_{|q|\delta s}}{\frac{1}{2}\rho\ell^4 U}$ Coefficient used in representing $M_{\delta s}$ as a function q

M_{rp} $M_{rp}' = \dfrac{M_{rp}}{\frac{1}{2}\rho\ell^5}$ Coefficient used in representing M as a function of the product rp

M_{rr} $M_{rr}' = \dfrac{M_{rr}}{\frac{1}{2}\rho\ell^5}$ Second order coefficient used in repres . z M as a function of r. First order coefficient is zero

M_{vp} $M_{vp}' = \dfrac{M_{vp}}{\frac{1}{2}\rho\ell^4}$ Coefficient used in representing M as a function of the product vp

M_{vr} $M_{vr}' = \dfrac{M_{vr}}{\frac{1}{2}\rho\ell^4}$ Coefficient used in representing M as a function of the product vr

M_{vv} $M_{vv}' = \dfrac{M_{vv}}{\frac{1}{2}\rho\ell^3}$ Second order coefficient used in representing M as a function of v

M_w $M_w' = \dfrac{M_w}{\frac{1}{2}\rho\ell^3 U}$ First order coefficient used in representing M as a function of w

$M_{w\eta}$ $M_{w\eta}' = \dfrac{M_{w\eta}}{\frac{1}{2}\rho\ell^3 U}$ First order coefficient used in representing M_w as a function of $(\eta-1)$

$M_{\dot{w}}$ $M_{\dot{w}}' = \dfrac{M_{\dot{w}}}{\frac{1}{2}\rho\ell^4}$ Coefficient used in representing M as a function of $\dot{w}$

$M_{|w|}$ $M_{|w|}' = \dfrac{M_{|w|}}{\frac{1}{2}\rho\ell^3 U}$ First order coefficient used in representing M as a function of w; equal to zero for symmetrical function

$M_{|w|q}$ $M_{|w|q}' = \dfrac{M_{|w|q}}{\frac{1}{2}\rho\ell^4}$ Coefficient used in representing M_q as a function of w

$M_{w|w|}$ $M_{w|w|}' = \dfrac{M_{w|w|}}{\frac{1}{2}\rho\ell^3}$ Second order coefficient used in representing M as a function of w

$M_{w|w|\eta}$ $M_{w|w|\eta}' = \dfrac{M_{w|w|\eta}}{\frac{1}{2}\rho\ell^3}$ First order coefficient used in representing $M_{w|w|}$ as a function of $(\eta-1)$

M_{ww} $M_{ww}' = \dfrac{M_{ww}}{\frac{1}{2}\rho\ell^3}$ Second order coefficient used in representing M as a function of w; equal to zero for symmetrical function

$M_{\delta b}$ $M_{\delta b}' = \dfrac{M_{\delta b}}{\frac{1}{2}\rho\ell^3 U^2}$ First order coefficient used in representing M as a function of δ_b

$M_{\delta s}$ $M_{\delta s}' = \dfrac{M_{\delta s}}{\frac{1}{2}\rho\ell^3 U^2}$ First order coefficient used in representing M as a function of δ_s

$M_{\delta s\eta}$ $M_{\delta s\eta}' = \dfrac{M_{\delta s\eta}}{\frac{1}{2}\rho\ell^3 U^2}$ First order coefficient used in representing $M_{\delta s}$ as a function of $(\eta-1)$

N	$N' = \dfrac{N}{\frac{1}{2}\rho \ell^3 U^2}$	Hydrodynamic moment component about z axis (yawing moment)
N_*	$N_*' = \dfrac{N_*}{\frac{1}{2}\rho \ell^3 U^2}$	Yawing moment when body angles (α, β) and control surface angles are zero
N_p	$N_p' = \dfrac{N_p}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing N as a function of p
$N_{\dot{p}}$	$N_{\dot{p}}' = \dfrac{N_{\dot{p}}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing N as a function of $\dot{p}$
N_{pq}	$N_{pq}' = \dfrac{N_{pq}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing N as a function of the product pq
N_{qr}	$N_{qr}' = \dfrac{N_{qr}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing N as a function of the product qr
N_r	$N_r' = \dfrac{N_r}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing N as a function of r
$N_{r\eta}$	$N_{r\eta}' = \dfrac{N_{r\eta}}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing N_r as a function of $(\eta - 1)$
$N_{\dot{r}}$	$N_{\dot{r}}' = \dfrac{N_{\dot{r}}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing N as a function of $\dot{r}$
$N_{r\|r\|}$	$N_{r\|r\|}' = \dfrac{N_{r\|r\|}}{\frac{1}{2}\rho \ell^5}$	Second order coefficient used in representing N as a function of r
$N_{\|r\|\delta r}$	$N_{\|r\|\delta r}' = \dfrac{N_{\|r\|\delta r}}{\frac{1}{2}\rho \ell^4 U}$	Coefficient used in representing $N_{\delta r}$ as a function of r
N_v	$N_v' = \dfrac{N_v}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing N as a function of v
$N_{v\eta}$	$N_{v\eta}' = \dfrac{N_{v\eta}}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing N_v as a function of $(\eta - 1)$
$N_{\dot{v}}$	$N_{\dot{v}}' = \dfrac{N_{\dot{v}}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing N as a function of $\dot{v}$
N_{vq}	$N_{vq}' = \dfrac{N_{vq}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing N as a function of the product vq
$N_{\|v\|r}$	$N_{\|v\|r}' = \dfrac{N_{\|v\|r}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing N_r as a function of v
$N_{v\|v\|}$	$N_{v\|v\|}' = \dfrac{N_{v\|v\|}}{\frac{1}{2}\rho \ell^3}$	Second order coefficient used in representing N as a function of v
$N_{v\|v\|\eta}$	$N_{v\|v\|\eta}' = \dfrac{N_{v\|v\|\eta}}{\frac{1}{2}\rho \ell^3}$	First order coefficient used in representing $N_{v\|v\|}$ as a function of $(\eta - 1)$

N_{vw}	$N_{vw}' = \dfrac{N_{vw}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing N as a function of the product vw
N_{wp}	$N_{wp}' = \dfrac{N_{wp}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing N as a function of the product wp
N_{wr}	$N_{wr}' = \dfrac{N_{wr}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing N as a function of the product wr
$N_{\delta r}$	$N_{\delta r}' = \dfrac{N_{\delta r}}{\frac{1}{2}\rho\ell^3 U^2}$	First order coefficient used in representing N as a function of δ_r
$N_{\delta r\eta}$	$N_{\delta r\eta}' = \dfrac{N_{\delta r\eta}}{\frac{1}{2}\rho\ell^3 U^2}$	First order coefficient used in representing $N_{\delta r}$ as a function of $(\eta-1)$
p	$p' = \dfrac{\rho\ell}{U}$	Angular velocity component about y axis relative to fluid (roll)
$\dot{p}$	$\dot{p}' = \dfrac{\dot{p}\ell^2}{U^2}$	Angular acceleration component about x axis relative to fluid
q	$q' = \dfrac{q\ell}{U}$	Angular velocity component about y axis relative to fluid (pitch)
$\dot{q}$	$\dot{q}' = \dfrac{\dot{q}\ell^2}{U^2}$	Angular acceleration component about y axis relative to fluid
r	$r' = \dfrac{r\ell}{U}$	Angular velocity component about z axis relative to fluid (yaw)
$\dot{r}$	$\dot{r}' = \dfrac{\dot{r}\ell^2}{U^2}$	Angular acceleration component about z axis relative to fluid
U	$U' = \dfrac{U}{U}$	Linear velocity of origin of body axes relative to fluid
u	$u' = \dfrac{u}{U}$	Component of U in direction of the x axis
$\dot{u}$	$\dot{u}' = \dfrac{\dot{u}\ell}{U^2}$	Time rate of change of u in direction of the x axis
u_c	$u_c' = \dfrac{u_c}{U}$	Command speed: steady value of ahead speed component u for a given propeller rpm when body angles (α, β) and control surface angles are zero. Sign changes with propeller reversal
v	$v' = \dfrac{v}{U}$	Component of U in direction of the y axis
$\dot{v}$	$\dot{v}' = \dfrac{\dot{v}\ell}{U^2}$	Time rate of change of v in direction of the y axis

w	$w' = \dfrac{w}{U}$	Component of U in direction of the z axis
$\dot{w}$	$\dot{w}' = \dfrac{\dot{w}\ell}{U^2}$	Time rate of change of w in direction of the z axis
W	$W' = \dfrac{W}{\frac{1}{2}\rho\ell^2 U^2}$	Weight, including water in free flooding spaces
x	$x' = \dfrac{x}{\ell}$	Longitudinal body axis; also the coordinate of a point relative to the origin of body axes
x_B	$x_B' = \dfrac{x_B}{\ell}$	The x coordinate of CB
x_G	$x_G' = \dfrac{x_G}{\ell}$	The x coordinate of CG
$\dot{x}_o$	$x_o' = \dfrac{x_o}{\ell}$	A coordinate of the displacement of CG relative to the origin of a set of fixed axes
X	$X' = \dfrac{X}{\frac{1}{2}\rho\ell^2 U^2}$	Hydrodynamic force component along x axis (longitudinal, or axial, force)
X_{qq}	$X_{qq}' = \dfrac{X_{qq}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing X as a function of q. First order coefficient is zero
X_{rp}	$X_{rp}' = \dfrac{X_{rp}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing X as a function of the product rp
X_{rr}	$X_{rr}' = \dfrac{X_{rr}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing X as a function of r. First order coefficient is zero
$X_{\dot{u}}$	$X_{\dot{u}}' = \dfrac{X_{\dot{u}}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of $\dot{u}$
X_{uu}	$X_{uu}' = \dfrac{X_{uu}}{\frac{1}{2}\rho\ell^2}$	Second order coefficient used in representing X as a function of u in the non-propelled case. First order coefficient is zero
X_{vr}	$X_{vr}' = \dfrac{X_{vr}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of the product vr
X_{vv}	$X_{vv}' = \dfrac{X_{vv}}{\frac{1}{2}\rho\ell^2}$	Second order coeffient used in representing X as a function of v. First order coefficient is zero
$X_{vv\eta}$	$X_{vv\eta}' = \dfrac{X_{vv\eta}}{\frac{1}{2}\rho\ell^2}$	First order coefficient used in representing X_{vv} as a function of $(\eta - 1)$
X_{wq}	$X_{wq}' = \dfrac{X_{wq}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of the product wq

X_{ww}	$X_{ww}' = \dfrac{X_{ww}}{\frac{1}{2}\rho \ell^2}$	Second order coefficient used in representing X as a function of w. First order coefficient is zero
$X_{ww\eta}$	$X_{ww\eta}' = \dfrac{X_{ww\eta}}{\frac{1}{2}\rho \ell^2}$	First order coefficient used in representing X_{ww} as a function of $(\eta-1)$
$X_{\delta b \delta b}$	$X_{\delta b \delta b}' = \dfrac{X_{\delta b \delta b}}{\frac{1}{2}\rho \ell^2 U^2}$	Second order coefficient used in representing X as a function of δ_b. First order coefficient is zero
$X_{\delta r \delta r}$	$X_{\delta r \delta r}' = \dfrac{X_{\delta r \delta r}}{\frac{1}{2}\rho \ell^2 U^2}$	Second order coefficient used in representing X as a function of δ_r. First order coefficient is zero
$X_{\delta r \delta r \eta}$	$X_{\delta r \delta r \eta}' = \dfrac{X_{\delta r \delta r \eta}}{\frac{1}{2}\rho \ell^2 U^2}$	First order coefficient used in representing $X_{\delta r \delta r}$ as a function of $(\eta-1)$
$X_{\delta s \delta s}$	$X_{\delta s \delta s}' = \dfrac{X_{\delta s \delta s}}{\frac{1}{2}\rho \ell^2 U^2}$	Second order coefficient used in representing X as a function of δ_s. First order coefficient is zero
$X_{\delta s \delta s \eta}$	$X_{\delta s \delta s \eta}' = \dfrac{X_{\delta s \delta s \eta}}{\frac{1}{2}\rho \ell^2 U^2}$	First order coefficient used in representing $X_{\delta s \delta s}$ as a function of $(\eta-1)$
y	$y' = \dfrac{y}{\ell}$	Lateral body axis; also the coordinate of a point relative to the origin of body axes
y_B	$y_B' = \dfrac{y_B}{\ell}$	The y coordinate of CB
y_G	$y_G' = \dfrac{y_G}{\ell}$	The y coordinate of CG
y_o	$y_o' = \dfrac{y_o}{\ell}$	A coordinate of the displacement of CG relative to the origin of a set of fixed axes
Y	$Y' = \dfrac{Y}{\frac{1}{2}\rho \ell^2 U^2}$	Hydrodynamic force component along y axis (lateral force)
Y_*	$Y_*' = \dfrac{Y}{\frac{1}{2}\rho \ell^2 U^2}$	Lateral force when body angles (α, β) and control surface angles are zero
Y_p	$Y_p' = \dfrac{Y_p}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing Y as a function of p
$Y_{\dot{p}}$	$Y_{\dot{p}}' = \dfrac{Y_{\dot{p}}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing Y as a function of $\dot{p}$
$Y_{p|p|}$	$Y_{p|p|}' = \dfrac{Y_{p|p|}}{\frac{1}{2}\rho \ell^4}$	Second order coefficient used in representing Y as a function of p

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Y_{pq} $Y_{pq}' = \dfrac{Y_{pq}}{\frac{1}{2}\rho \ell^4}$ Coefficient used in representing Y as a function of the product pq

Y_{qr} $Y_{qr}' = \dfrac{Y_{qr}}{\frac{1}{2}\rho \ell^4}$ Coefficient used in representing Y as a function of the product qr

Y_r $Y_r' = \dfrac{Y_r}{\frac{1}{2}\rho \ell^3 U}$ First order coefficient used in representing Y as a function of r

$Y_{r\eta}$ $Y_{r\eta}' = \dfrac{Y_{r\eta}}{\frac{1}{2}\rho \ell^3 U}$ First order coefficient used in representing Y_r as a function of $(\eta-1)$

$Y_{\dot{r}}$ $Y_{\dot{r}}' = \dfrac{Y_{\dot{r}}}{\frac{1}{2}\rho \ell^4}$ Coefficient used in representing Y as a function of $\dot{r}$

$Y_{|r|\delta r}$ $Y_{|r|\delta r}' = \dfrac{Y_{|r|\delta r}}{\frac{1}{2}\rho \ell^3 U}$ Coefficient used in representing $Y_{\delta r}$ as a function of r

Y_v $Y_v' = \dfrac{Y_v}{\frac{1}{2}\rho \ell^2 U}$ First order coefficient used in representing Y as a function of v

$Y_{v\eta}$ $Y_{v\eta}' = \dfrac{Y_{v\eta}}{\frac{1}{2}\rho \ell^2 U}$ First order coefficient used in representing Y_v as a function of $(\eta-1)$

$Y_{\dot{v}}$ $Y_{\dot{v}}' = \dfrac{Y_{\dot{v}}}{\frac{1}{2}\rho \ell^3}$ Coefficient used in representing Y as a function of $\dot{v}$

Y_{vq} $Y_{vq}' = \dfrac{Y_{vq}}{\frac{1}{2}\rho \ell^3}$ Coefficient used in representing Y as a function of the product vq

$Y_{v|r|}$ $Y_{v|r|}' = \dfrac{Y_{v|r|}}{\frac{1}{2}\rho \ell^3}$ Coefficient used in representing Y_v as a function of r

$Y_{v|v|}$ $Y_{v|v|}' = \dfrac{Y_{v|v|}}{\frac{1}{2}\rho \ell^2}$ Second order coefficient used in representing Y as a function of v

$Y_{v|v|\eta}$ $Y_{v|v|\eta}' = \dfrac{Y_{v|v|\eta}}{\frac{1}{2}\rho \ell^2}$ First order coefficient used in representing $Y_{v|v|}$ as a function of $(\eta-1)$

Y_{vw} $Y_{vw}' = \dfrac{Y_{vw}}{\frac{1}{2}\rho \ell^2}$ Coefficient used in representing Y as a function of the product vw

Y_{wp} $Y_{wp}' = \dfrac{Y_{wp}}{\frac{1}{2}\rho \ell^3}$ Coefficient used in representing Y as a function of the product wp

Y_{wr} $Y_{wr}' = \dfrac{Y_{wr}}{\frac{1}{2}\rho \ell^3}$ Coefficient used in representing Y as a function of the product wr

$Y_{\delta r}$ $Y_{\delta r}' = \dfrac{Y_{\delta r}}{\frac{1}{2}\rho \ell^2 U^2}$ First order coefficient used in representing Y as a function of δr

$Y_{\delta r\eta}$ $Y_{\delta r\eta}' = \dfrac{Y_{\delta r\eta}}{\frac{1}{2}\rho \ell^2 U^2}$ First order coefficient used in representing $Y_{\delta r}$ as a function of $(\eta-1)$

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z	$z' = \dfrac{z}{\ell}$	Normal body axis; also the coordinate of a point relative to the origin of body axes
z_B	$z_B' = \dfrac{z_B}{\ell}$	The z coordinate of CB
z_G	$z_G' = \dfrac{z_G}{\ell}$	The z coordinate of CG
z_o	$z_o' = \dfrac{z_o}{\ell}$	A coordinate of the displacement of CG relative to the origin of a set of fixed axes
Z	$Z' = \dfrac{Z}{\frac{1}{2}\rho\ell^2 U^2}$	Hydrodynamic force component along z axis (normal force)
Z_*	$Z_*' = \dfrac{Z_*}{\frac{1}{2}\rho\ell^2 U^2}$	Normal force when body angles (α, β) and control surface angles are zero
Z_{pp}	$Z_{pp}' = \dfrac{Z_{pp}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing Z as a function of p. First order coefficient is zero
Z_q	$Z_q' = \dfrac{Z_q}{\frac{1}{2}\rho\ell^3 U}$	First order coefficient used in representing Z as a function of q
$Z_{q\eta}$	$Z_{q\eta}' = \dfrac{Z_{q\eta}}{\frac{1}{2}\rho\ell^3 U}$	First order coefficient used in representing Z_q as a function of $(\eta-1)$
$Z_{\dot{q}}$	$Z_{\dot{q}}' = \dfrac{Z_{\dot{q}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Z as a function of $\dot{q}$
$Z_{|q|\delta s}$	$Z_{|q|\delta s}' = \dfrac{Z_{|q|\delta s}}{\frac{1}{2}\rho\ell^3 U}$	Coefficient used in representing $Z_{\delta s}$ as a function of q
Z_{rp}	$Z_{rp}' = \dfrac{Z_{rp}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Z as a function of the product rp
Z_{rr}	$Z_{rr}' = \dfrac{Z_{rr}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing Z as a function of r. First order coefficient is zero
Z_w	$Z_w' = \dfrac{Z_w}{\frac{1}{2}\rho\ell^2 U}$	First order coefficient used in representing Z as a function of w
$Z_{w\eta}$	$Z_{w\eta}' = \dfrac{Z_{w\eta}}{\frac{1}{2}\rho\ell^2 U}$	First order coefficient used in representing Z_w as a function of $(\eta-1)$
$Z_{\dot{w}}$	$Z_{\dot{w}}' = \dfrac{Z_{\dot{w}}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Z as a function of $\dot{w}$
$Z_{|w|}$	$Z_{|w|}' = \dfrac{Z_{|w|}}{\frac{1}{2}\rho\ell^2 U}$	First order coefficient used in representing Z as a function of w; equal to zero for symmetrical function
$Z_{w|q|}$	$Z_{w|q|}' = \dfrac{Z_{w|q|}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Z_w as a function of q

$Z_{w|w|}$ $\quad Z_{w|w|}' = \dfrac{Z_{w|w|}}{\frac{1}{2}\rho\ell^2}$ $\quad$ Second order coefficient used in representing Z as a function of w

$Z_{w|w|\eta}$ $\quad Z_{w|w|\eta}' = \dfrac{Z_{w|w|\eta}}{\frac{1}{2}\rho\ell^2}$ $\quad$ First order coefficient used in representing $Z_{w|w|}$ as a function of $(\eta-1)$

Z_{ww} $\quad Z_{ww}' = \dfrac{Z_{ww}}{\frac{1}{2}\rho\ell^2}$ $\quad$ Second order coefficient used in representing Z as a function of w; equal to zero for symmetrical function

$Z_{\delta b}$ $\quad\bullet\quad Z_{\delta b}' = \dfrac{Z_{\delta b}}{\frac{1}{2}\rho\ell^2 U^2}$ $\quad$ First order coefficient used in representing Z as a function of δ_b

$Z_{\delta s}$ $\quad Z_{\delta s}' = \dfrac{Z_{\delta s}}{\frac{1}{2}\rho\ell^2 U^2}$ $\quad$ First order coefficient used in representing Z as a function of δ_s

$Z_{\delta s\eta}$ $\quad Z_{\delta s\eta}' = \dfrac{Z_{\delta s\eta}}{\frac{1}{2}\rho\ell^2 U^2}$ $\quad$ First order coefficient used in representing $Z_{\delta s}$ as a function of $(\eta-1)$

α $\quad$ Angle of attack

β $\quad$ Angle of drift

δ_b $\quad$ Deflection of bowplane or sailplane

δ_r $\quad$ Deflection of rudder

δ_s $\quad$ Deflection of sternplane

η $\quad$ The ratio $\dfrac{u_c}{U}$

θ $\quad$ Angle of pitch

ψ $\quad$ Angle of yaw

ϕ $\quad$ Angle of roll

a_i, b_i, c_i $\quad$ Sets of constants used in the representation of propeller thrust in the axial equation

xi

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(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Goodyear Aerospace Corp., Akron, Ohio	Unclassified
	2b. GROUP

3. REPORT TITLE

ADVANCED SUBMARINE SYSTEMS EQUATIONS STUDY

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Final Report

5. AUTHOR(S) *(First name, middle initial, last name)*

Brenton R. Groves, Ph.D.
James T. Dorsey
Dennis Tucker

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
November 1969	109	18

8a. CONTRACT OR GRANT NO	9a. ORIGINATOR'S REPORT NUMBER(S)
N61339-68-C-0050	
b. PROJECT NO. 7843	GER-14448
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	NAVTRADEVCEN 68-C-0050-1

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	U.S. Naval Training Device Center Orlando, Fla.

13. ABSTRACT

The goal of this study was to determine the optimum sets of equations to be used with two general types of submarine trainers. The equations of motion used were those developed by the Naval Ship Research and Development Center. They were reduced into two sets for use in training simulators, one for a basic submerged control maneuvering trainer and one for an advanced submerged control simulator to provide highly realistic ship control training through the full range of normal and casualty conditions.

The report outlines a general-purpose digital computer program, following the NSRDC standard equations, written in FORTRAN. Integration methods for digital simulators are discussed. A number of programs for testing the degree of simulation of a digital simulator program are given.

Use of these programs as applied to submarine simulation is shown with two sets of equations which eliminate 73 of the 139 coefficients used in the original NSRDC equations. The mathematical model for a submarine simulator using a very small computer is developed. A discussion of near-surface operation and wave-generation is followed by the general requirements for determination of the simulation requirements for training as opposed to research.

DD FORM 1473 (NOV 65)

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Simulators						
Training Devices						
Digital Simulation						
Submarine Simulators						
Equations of Motion						
Mathematical Models						
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